THE UNIVERSITY OF ALBERTA

RELEASE FORM

NAME OF AUTHOR [illegible]

TITLE OF THESIS [illegible]

[illegible]

DEGREE FOR WHICH THESIS WAS PRESENTED [illegible]

YEAR THIS DEGREE GRANTED [illegible]

[illegible]

THE UNIVERSITY OF ALBERTA

RELEASE FORM

NAME OF AUTHOR MILE KOSOVAC

TITLE OF THESIS SUBOPTIMAL CONTROL OF A SYNCHRONOUS GENERATOR WITH TWO FIELD WINDINGS

DEGREE FOR WHICH THESIS WAS PRESENTED MASTER OF SCIENCE

YEAR THIS DEGREE GRANTED 1973

THE UNIVERSITY OF ALBERTA

SUBOPTIMAL CONTROL OF A SYNCHRONOUS GENERATOR WITH TWO FIELD WINDINGS

by

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A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES AND RESEARCH IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF MASTER OF SCIENCE

DEPARTMENT OF ELECTRICAL ENGINEERING

EDMONTON, ALBERTA

SPRING 1973

THE UNIVERSITY OF ALBERTA

FACULTY OF GRADUATE STUDIES AND RESEARCH

The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research for acceptance, a thesis entitled Suboptimal Control of Synchronous Generator with Two Field Windings submitted by Mile Kosovac in partial fulfillment of the requirements for the degree of Master of Science.

ABSTRACT

The operation of synchronous generators with only a direct-axis field winding is severely limited at leading power factor. A generator with an additional winding on the quadrature axis, provided with suitable control, shows considerable advantages over the conventional generator.

This thesis is concerned in the first part with a comparison between a conventional wound-rotor and a divided-winding-rotor machine, and with the advantage gained by the use of the divided-winding rotor machine in the system. In the second part the feasibility of using a suboptimal control law based on the nonlinear differential equations of a divided-winding rotor synchronous generator is discussed. Numerical results are presented for single generator-infinite bus problems implementing such control.

ACKNOWLEDGEMENT

The author wishes to acknowledge his indebtedness to Dr. D. H. Kelly whose guidance throughout this project was invaluable.

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CHAPTER 1

INTRODUCTION

1.1 Background

The annual Canadian electricity demand has increased at a rate of five percent per year. There seems little doubt that this rate of increase will continue in the future, perhaps it may even accelerate. For the post war period it was recognized that rapid economic growth, necessitates the development of large power systems and networks of large transmission capacities. As power systems grow,they become interconnected in accordance with the development plan for the economic zones in which they are located, and afterwards power systems in neighboring economic zones will be interconnected. Interconnections have many advantages. Some of the advantages are caused by diversity (daily, seasonal and annual load diversity), staggered installation of facilities, nonsimultaneous failures, and scheduled maintenance. Thus, a fault which disturbs the stability of the system affects the whole system and not only the part in which the fault occurs. Without adequate coordination, the interconnected system may suffer a catastrophic failure. Experience indicates that instability or system collapse, usually develops after a period of time as a result of automatic control operations. In a transient stability study the action of

regulators, governors, relays and other automatic equipment should be recognized. For the high reliability required of a modern power system, stability must be ensured and system control must be automatic. To meet these high quality demands the use of automatic control theories has been necessary.

1.2 Power System Stability

The stability of synchronous machines has been of concern to the utility industry since its early days, because the quality of every system, not only power systems, is determined by its stability. To say that a power system is stable implies that the energy produced by the generators is transmitted to the consumers by keeping voltage at the consumers terminals and the frequency in the system in certain ranges. Although this capability has allowed the treatment of the stability phenomenon as one overall problem, still it is useful to continue examining the nature of the system behavior under three classifications of stability, i.e. steady-state, transient and dynamic.

Steady-state stability

This type of stability is characterised by the ability of a system to maintain gradually attained power transfer over the system without loss of stability. Here a dis-

tinction could be made between the cases of unregulated and regulated systems. The instability of the unregulated generator is reached at the peak of the steady-state power angle curve. The regulated system increases the steady-state limit. This stability may be called an artificial stability by which is understood an operating condition of a synchronous generator, which can only be maintained with the aid of automatic control. If the automatic control fails the generator goes out of synchronism.

Transient stability

While steady-state stability is the ability of the system to operate stably at a given fixed loading and transmission system conditions, transient stability refers to the ability of the system to maintain stability in the presence of a sudden large change in load caused by system switching or by a fault. Generally in a power system a transient condition occurs whenever it is undergoing a quick change from one state to another, and the variables defining the state differ greatly from their values in the normal steady state (i.e. angular velocity, angular acceleration). The duration of a transient condition is short but transient stability[25] is of great importance for systems. The overall merit of an electrical system is almost completely determined by its transient behavior.

A system which lacks stability in the transient state has no practical value.

Dynamic stability refers to the ability of the system to ensure very rapid damping of small oscillations occurring during normal operation (because a succession of small disturbances, for example, changes of load, occur continually, so that, strictly speaking, the system never operates in a steady-state) and to make quite certain that self-oscillation, i.e., oscillation with increasing amplitude, cannot arise. In effect dynamic stability is stability in the sense of small signal control system stability.

1.3 Divided-Winding-Rotor Power System Stability

The normal range of stable operation of a generator under loaded conditions can be extended by the action of a voltage regulator on a direct-axis field winding. Growing high-voltage transmission networks and increasing use of high-voltage cables produce excess reactive power which cannot economically be fully compensated by parallel reactors, and at a time of low power consumption a regulator acting on a direct-axis field winding has little effect. An additional winding on the quadrature axis, provided with a suitable control can increase the stable operating region: that is, cause an

improvement of the reactive power-absorption limit at any load condition especially under conditions of light load.[14] Another important aspect of the additional field winding is its ability to maintain the air-gap flux at a high level, following a fault, and hence to maintain a large voltage behind transient reactance. The result of this, is high transient-power transfer and good transient-stability performance.

1.4 Stability Control

Rapid automatic control of generator excitation and compensation for line inductive reactance (or in general changes in the network) are two principal methods of improving the stability limit of a power system. Both methods act to increase power transfer between machines.

The basic equation describing the dynamic (electro-mechanical) performance of a generator in a large network is

$$T_j\, p^2\delta + K_d\, p\delta = P_t - P_{el} \tag{1.1}$$

This is called the swing equation in which P_t is the turbine power imput and P_{el} is the electrical power output of the generator. When a generator is subjected to a sudden change in loading there is an imbalance between the

mechanical imput and the electrical output. The prime-mover cannot act instantly as it has an appreciable time-lag. By decreasing the governor time constant and by the use of high governor gain the stability limit can be increased. Another possibility of control arises from a study of the power output of a synchronous generator.

The power in a steady-state

$$P_{el} = \frac{E_d V}{(X_d + X_e)} \sin\delta \tag{1.2}$$

or in the transient case

$$P_{el} = \frac{E_d' V}{(X_d' + X_e)} \sin\delta \tag{1.3}$$

where E_d and E_d' are the voltages behind the synchronous reactance X_d and the transient reactance $X_{d'}$, respectively. By reducing the transfer reactance between synchronous generators (the new transfer reactance is $X_d + X_e - X_c$) i.e., by increasing the power transfer, either series or parallel capacitance could theoretically be used, but in practice only series capacitors have been installed, owing to the capital cost and technical difficulty of making capacitors of suitable rating to operate at line voltages of the order of 500 KV. Inserting and removing a series capacitors in a transmission line can remove the oscillatory

transient due to a fault. This may be done by controllers of an elementary type which alter the transmission properties in a nearly optimal fashion as a function of the system states. Some recent papers[16,23] are concerned with the application of optimal control theory in using series-capacitor switching.

Another method which also uses changes in the network is resistor braking. This device is a bank of resistors, located near a large generating plant, connected in shunt with the three-phase bus through a suitable switch that is normally open. The connection of the resistor is initiated by an increase of rotor kinetic energy over a suitable threshold value.[30]

The first method of rapid control of generator excitation has been comprehensively studied in the technical literature.[5,6,10,29] Examining equations (1.2) and (1.3) it can be seen that by controlling the voltage E_d behind the synchronous reactance, in steady state condition, or by controlling the voltage E_d' behind the transient reactance, electrical power output can be controlled. Considering not only the simple one-machine infinite bus system but also the multi-machine system it is found that by raising internal voltages the power that can be transmitted between any two machines or groups of machines can

be increased. The dependance of power on the internal voltage of the generator is shown in Fig. 1, 2, 3. Fig. 1 is the condition when the field voltage is held constant (non-controlled case).

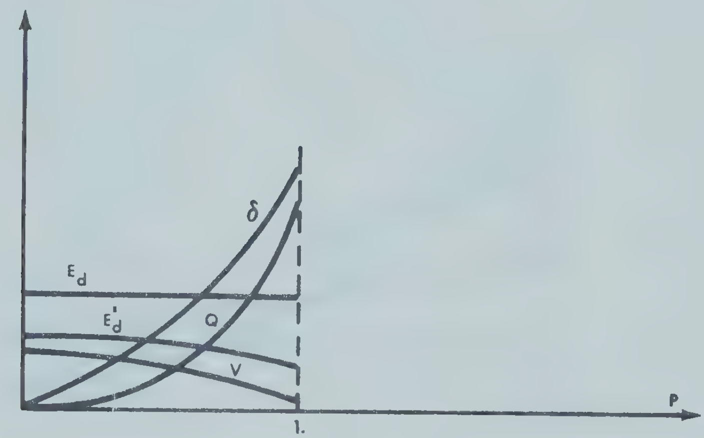

Fig. 1

For the first controlled case (Fig. 2) the transient voltage E_d' (which is proportional to field flux linkages and for some conditions to the field voltage) is held constant. The maximum power is appreciably greater.

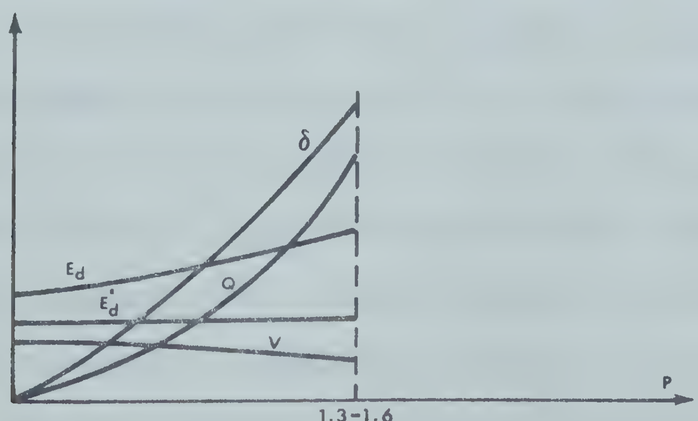

Fig. 2

Quick-response excitation, which is able to keep the terminal voltage constant for all values of load will ensure an almost ideally controlled power limit.

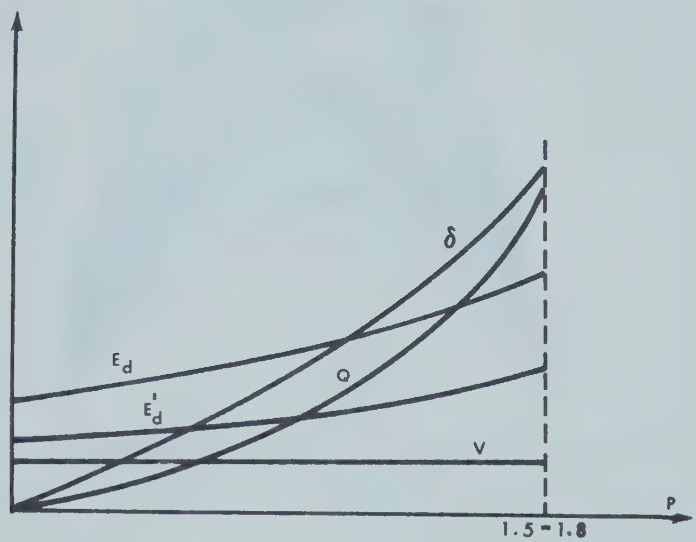

Fig. 3

By changing the internal machine voltage, power output is changed. Taking some values of transient voltages and considering them as constant for some specific region a family of power curves can be drawn as in Fig. 4.

Comparing the three operating regions of a synchronous generator from Fig. 1, 2, 3 with the following diagram, one more feature of excitation can be noted. The power limit is increased beyond 90°, which is a very important feature for transient stability, since the merit of transient stability is based on the first swing (if the system is stable during the first swing it is

stable afterwards).

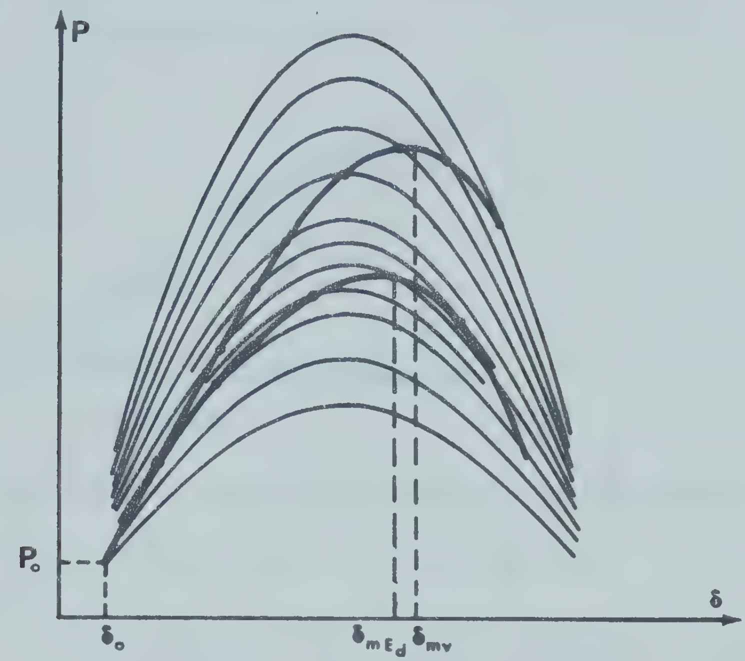

Fig. 4

As already mentioned in part 1.3 during low power consumption, the generator, because of increased charging current in the high-voltage transmission network, often has a leading power factor and may need to operate beyond the normal steady-state stability limit (which is determined by its capability for absorption of reactive power). Regulation on the normal direct-axis field winding can not extend the stability range. This failure of one field excitation is eliminated by two field excitation. The improvement in stability can be seen from Fig. 5.

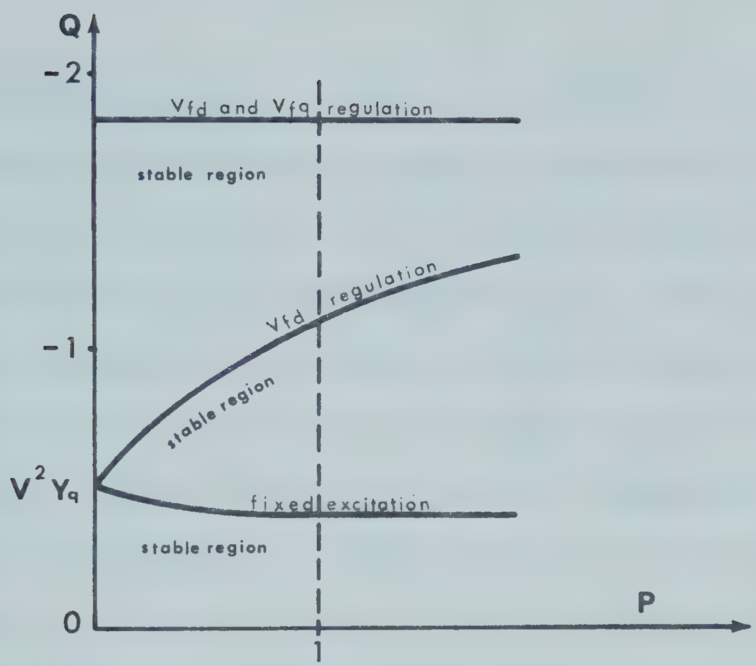

Fig. 5

Originally the main function of an automatic excitation regulator, was to maintain, during normal operation, a specified voltage across the system bus-bars and supply network. The function of excitation control has since been broadened considerably. Control of the excitation under fault condition or immediately after fault makes it possible to improve the stability of stations operating in parallel, in order to stabilize the load, prevent a sudden fall of voltage and ensure satisfactory starting of induction motors. All these lead to some specific requirements which must be satisfied by an automatic excitation regulator. The operating capacity of the transmission system should be utilised up to its limit. A good margin of transient stability and good

damping of hunting should be provided. Switching of the number of transmission lines, increases or decreases of any kind of load must not deteriorate the stability. This can be attained if the exciter has a high sensitivity, quick response and a high ceiling voltage (positive and negative ceiling have to be implemented, because at some moment a rapid fall of voltage is required). To do this, the methods of measuring the quantities, which actuate the automatic control must be simple and reliable. The time delay due to inertia in the control device and in all components of the excitation system must be small. It has been shown[8,10,27,29] that the stability of a synchronous machine is improved when the regulators respond not only to the variations of the controlling signals (voltage, current and angle) but also to the derivatives of these variations. Many recently published papers[15,31] dealing with the application of the optimal control theory to power system have proved this. Actually the use of velocity and accelerating signals and their combination specially through an optimal strategy is most effective for system stabilization.

1.5 Scope of Thesis

In earlier studies[9] of the synchronous machine with two-axis excitation it was proposed that the machine

would normally run with excitation on one field winding only, and the control on the second winding used whenever a severe disturbance occurs. After steady-state conditions have been reached, the quadrature excitation would be removed, and normal operation restored. In order to further improve the performance of a synchronous machine, not only during the transient period but in all operating conditions, continuous excitation with independent feedback system control is required for each winding, with independent feedback systems. The important feature of these two independently controlled windings is that the active power P is controlled by the quadrature-axis field which is usually called the "torque winding" and the reactive power is controlled by the direct-axis field which is usually called the "reactive winding". In studies of the d.w.r. machine many references[23,25] deal with a voltage regulator on the direct axis and an angle regulator on the quadrature axis. Some references[10,14] give a detailed study of d.w.r. machines with various regulators on the direct and quadrature axes (voltage regulator, angle regulator, proportional voltage regulator, proportional angle regulator, derivative angle regulator). The scope of this thesis is to find a time optimal control for d.w.r. machine as a function of the states. A one-

machine infinite bus system is used. The optimal strategy is formulated through a third order equation (derived from the nonlinear one-machine infinite bus model). The time optimal strategy gives bang-bang control. Proportional control as a link between theoretical and practical solutions is investigated. After that the combination of a proportional signal as a stabilizing signal and simple voltage and angle regulators is studied.

CHAPTER 2

MATHEMATICAL MODEL

2.1 Synchronous Generator Equations

A three-phase non salient-pole machine with amortisseur windings is considered. Iron loss is included in the amortisseur windings. Saturation is neglected. Such a machine has seven windings. On the rotor there are two field windings and two amortisseurs windings. On the stator there are three stator phase windings. The mathematical representation of the synchronous generator employs Park's[28] equations. The equations which give a complete description of the transient and steady-state operation of a d.w.r. synchronous generator are summarized in the following way:

Direct axis armature flux linkage

$$\psi_d = x_{afd}\, i_{fd} + x_{akd}\, i_{kd} - x_d\, i_d \tag{2.1}$$

Quadrature axis armature flux linkage

$$\psi_q = x_{afq}\, i_{fq} + x_{akq}\, i_{kq} - x_q\, i_q \tag{2.2}$$

Direct axis field flux linkage

$$\psi_{fd} = x_{ffd}\, i_{fd} + x_{fkd}\, i_{kd} - x_{afd}\, i_d \tag{2.3}$$

Quadrature axis field flux linkage

$$\psi_{fq} = x_{ffq}\, i_{fq} + x_{fkq}\, i_{kq} - x_{afq}\, i_q \tag{2.4}$$

Direct axis amortisseur flux linkage

$$\psi_{fkd} = x_{fkd}\, i_{fkd} + x_{kkd}\, i_{kd} - x_{akd}\, i_d \tag{2.5}$$

Quadrature axis amortisseur flux linkage

$$\psi_{fkq} = x_{fkq}\, i_{fq} + x_{kkq}\, i_{kq} - x_{akq}\, i_q \tag{2.6}$$

Direct axis armature voltage

$$e_d = \frac{1}{\omega_o} p\psi_d - Ri_d - \frac{\omega}{\omega_o}\psi_q \tag{2.7}$$

Quadrature axis armature voltage

$$e_q = \frac{1}{\omega_o} p\psi_q - Ri_q + \frac{\omega}{\omega_o}\psi_d \tag{2.8}$$

Direct axis field voltage

$$v_{fd} = \frac{1}{\omega_o} p\psi_{fd} + r_{fd}\, i_{fd} \tag{2.9}$$

Quadrature axis field voltage

$$v_{fq} = \frac{1}{\omega_o} p\psi_{fq} + r_{fq}\, i_{fq} \tag{2.10}$$

Direct axis amortisseur voltage equation

$$o = \frac{1}{\omega_o} p\psi_{fkd} + r_{kd}\, i_{kd} \tag{2.11}$$

Quadrature axis amortisseur voltage equation

$$o = \frac{1}{\omega_o} p\psi_{fkq} + r_{kq} i_{kq} \tag{2.12}$$

The self and mutual reactances used in the above equations are expressed as the sum of the mutual inductance and the leakage inductance.

$$x_d = x_{ad} + x_{al} \tag{2.13}$$

$$x_q = x_{aq} + x_{al} \tag{2.14}$$

$$x_{ffd} = x_{ad} + x_{fl} \tag{2.15}$$

$$x_{ffq} = x_{aq} + x_{fl} \tag{2.16}$$

$$x_{kkd} = x_{ad} + x_{kl} \tag{2.17}$$

$$x_{kkq} = x_{aq} + x_{kl} \tag{2.18}$$

Mechanical equation of motion

A synchronous generator is an oscillatory system. A study of this system deals with electro-mechanical processes, which are associated with the displacement of a heavy rotor and the change of stored magnetic energy. The dynamics of the synchronous generator with the speed damping term included are represented by the differential equation

$$T_j\, p^2\delta + K_d\, p\delta = T_t - T_{el} \tag{2.19}$$

The stored energy which is characterised by the term T_j depends on the machine instantaneous rotor angular velocity (or instantaneous frequency) and for a given machine T_j may be expressed as

$$T_j = T_{jo}\frac{\omega_r}{\omega_o} \tag{2.20}$$

For this study the constant value of T_j, i.e., T_{jo} will be used (at synchronous speed ω_o). Actually transient frequency excursions go up to ±5% of the synchronous frequency. Neglecting this gives results which are safe, but sometimes pessimistic.

Electrical torque at air gap

The term T_{el} on the right hand side of equation (2.19) is the electromagnetic torque which has to match the mechanical turbine torque T_t and thus prevent torque differentials that pull the generator out of synchronism. The electromagnetic torque is expressed as:

$$T_{el} = \psi_d\, i_q - \psi_q\, i_d \tag{2.21}$$

The first component is due to the interaction between the direct-axis flux and the quadrature-axis current, and the second component is due to the interaction between the

quadrature-axis flux and the direct-axis current.

2.2 Single-Machine Infinite Bus System

The present study is concerned with the system of Fig. 2.1 in which a generator, connected to an infinite bus through a two circuit line is equipped with a voltage regulator on one field winding (direct axis) and an angle regulator on the other (quadrature axis).

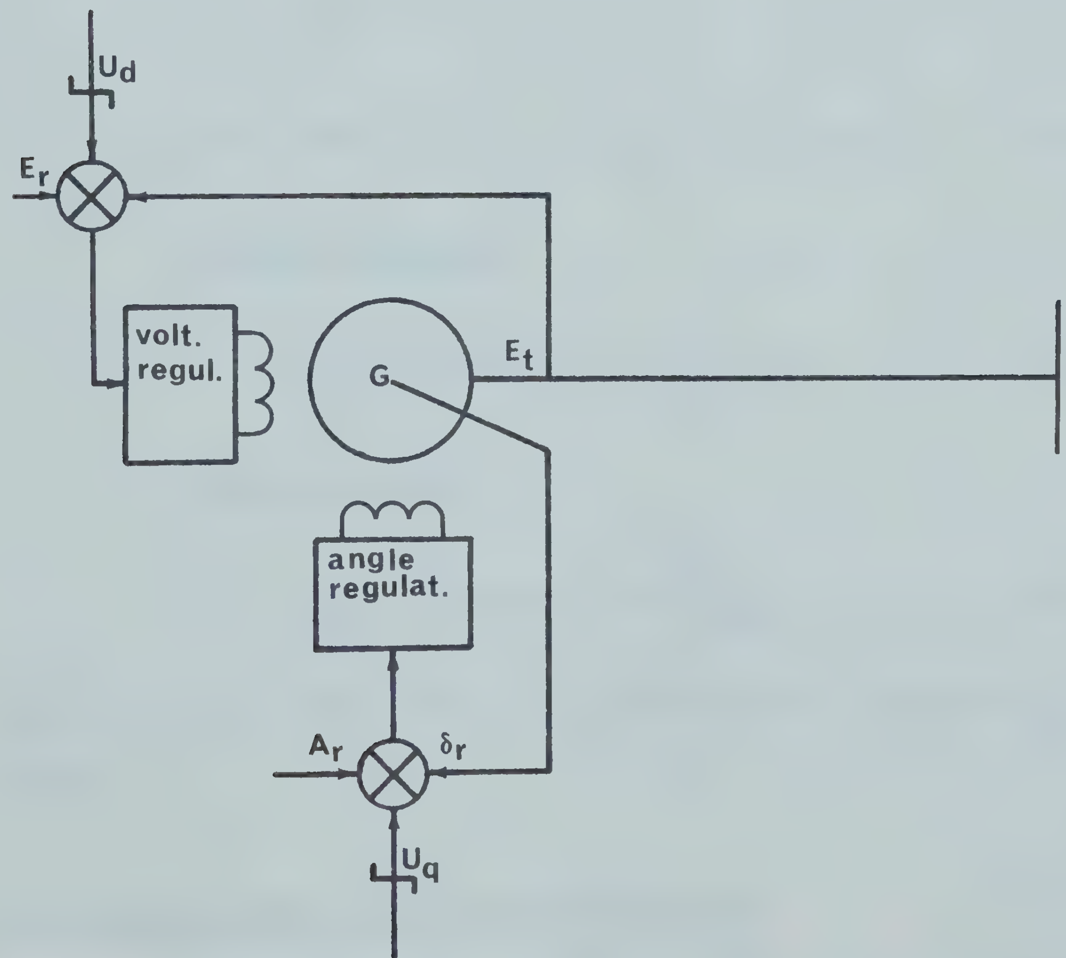

Fig. 2.1

To complete the mathematical model of this system equations of the transmission line and the regulators have to be added to the equations of the synchronous generator.

Transmission system

The transmission system is represented by lumped series inductance and resistance in Park's reference frame. The infinite bus system is characterised by constant frequency and voltage and zero impedance.

$$e_d = v \sin\delta + R_e \, i_d - \frac{\omega}{\omega_o} x_e \, i_q + L_e \, p i_d \qquad (2.22)$$

$$e_q = v \cos\delta + R_e \, i_q + \frac{\omega}{\omega_o} x_e \, i_d + L_e \, p i_q \qquad (2.23)$$

Voltage regulator

$$\frac{V_{fd}}{E_r - E_t + u(t)} = \frac{K_v}{1 + \tau_v \, p} \qquad (2.24)$$

Angle regulator

$$\frac{V_{fq}}{A_r + \delta + u(t)} = \frac{K_A}{1 + \tau_a \, p} \qquad (2.25)$$

where u(t) is a bang-bang or a proportional stabilizing signal.

2.3 Steady-State Phasor Representation

Phasor diagrams are very useful in the theory of

electric machines. They give relations between the variables as the operating conditions change. But the distinction between the phasor diagram and stability is an important one. The phasor diagram tells nothing about stability. To get a better picture of the d.w.r. generator the conventional phasor diagram for a generator is drawn.

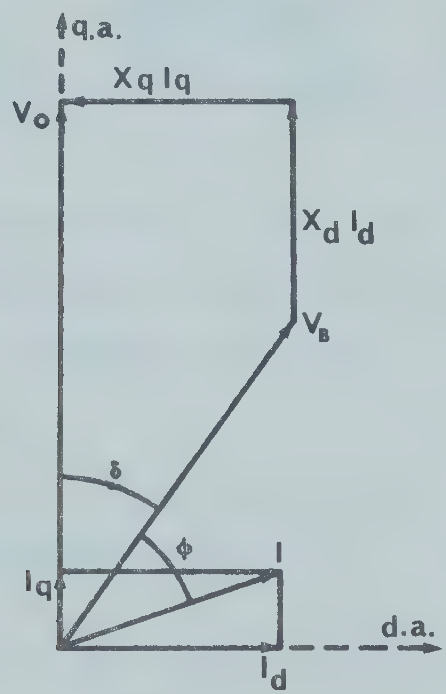

Fig. 2.2

Fig. 2.2 is a conventional phasor diagram (excitation on direct axis only) at a lagging power factor. Fig. 2.3 shows the diagram for the same operational condition when the machine has a quadrature winding control which can hold δ at a specific value.

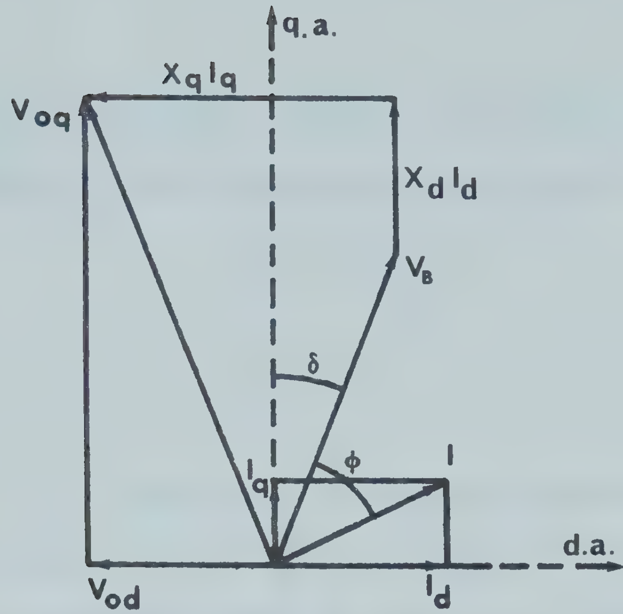

Fig. 2.3

It was mentioned in part 1.5 that one of the advantages of the d.w.r. generator is that the active power P is controlled by the quadrature axis field. It will be confirmed in the following.

From Fig. 2.3 it can be seen that V_o is composed of two components V_{od} and V_{oq}.

$$V_{od} = V_B \sin\delta - X_q I_q$$

$$V_{oq} = V_B \cos\delta + X_d I_d \qquad (2.26)$$

The excitation voltage components V_{od} and V_{oq} (behind synchronous impedances X_d and X_q respectively) are directly proportional to the currents I_{fq}, I_{fd} respectively.

$$V_{od} = x_{aq} I_{fq}$$

$$V_{oq} = x_{ad} I_{fd} \qquad (2.27)$$

By resolving the stator current into two components I_p and I_v the power of the synchronous generator is

$$P_o = V_B I_p$$

$$Q_o = V_B I_v \tag{2.28}$$

where I_p is the component of stator current in the phase with bus voltage V_B and the I_v component of stator current lags the bus voltage by 90° (Fig. 2.4).

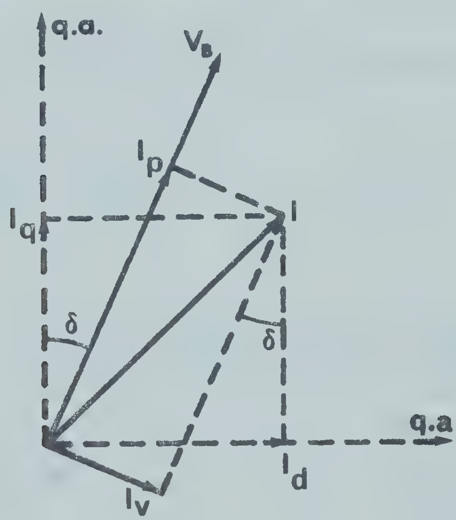

Fig. 2.4

The relation between two sets of current components I_p, I_v and I_d, I_q are:

$$I_p = I_q \cos\delta + I_d \sin\delta$$

$$I_v = I_d \cos\delta - I_q \sin\delta \tag{2.29}$$

From the equations 2.23 and 2.22 expressions for I_q and I_d are:

$$I_q = \frac{x_{aq}}{x_q} I_{fq} + \frac{V_B}{x_q} \sin\delta$$

$$I_d = \frac{x_{ad}}{x_d} I_{fd} - \frac{V_B}{x_d} \cos\delta \qquad (2.30)$$

Substituting 2.30 into 2.29 and then into 2.28 expressions for power of generator are:

$$P_o = \frac{x_{aq}}{x_q} I_{fq} V_B \cos\delta + \frac{V_B^2}{x_q} \sin\delta \cos\delta + \frac{x_{ad}}{x_d} I_{fd} V_B \sin\delta - \frac{V_B^2}{x_d} \cos\delta \sin\delta$$

$$Q_o = \frac{x_{ad}}{x_d} I_{fd} V_B \cos\delta - \frac{V_B^2}{x_d} \cos^2\delta - \frac{x_{aq}}{x_q} I_{fq} V_B \sin\delta - \frac{V_B^2}{x_q} \sin^2\delta \qquad (2.31)$$

If the angle δ is held constant at zero value the expressions for the power of a synchronous generator are:

$$P_o = \frac{x_{aq}}{x_q} I_{fq} V_B \qquad (2.32)$$

$$Q_o = \frac{x_{ad}}{x_d} I_{fd} V_B - \frac{V_B^2}{x_d} \qquad (2.33)$$

The active power P_o is controlled only by the quadrature axis field winding, while the reactive power Q_o is controlled by the direct axis field current.

2.4 The State Space Equations of the Single-Generator Infinite Bus Case

Combining equations 2.1-2.12 and 2.22-2.23 gives the form

$$\begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} \begin{bmatrix} pi_{fd} \\ pi_{kd} \\ pi_d \end{bmatrix} =$$

$$= \begin{bmatrix} 0 & 0 & B_{13} & \frac{\omega}{\omega_o} B_{14} & \frac{\omega}{\omega_o} B_{15} & \frac{\omega}{\omega_o} B_{16} \\ B_{21} & 0 & 0 & 0 & 0 & 0 \\ 0 & B_{32} & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} i_{fd} \\ i_{kd} \\ i_d \\ i_{fq} \\ i_{kq} \\ i_q \end{bmatrix} + \begin{bmatrix} V_d \\ V_{fd} \\ 0 \end{bmatrix} \qquad (2.34)$$

$$\begin{bmatrix} E_{11} & E_{12} & E_{13} \\ E_{21} & E_{22} & E_{23} \\ E_{31} & E_{32} & E_{33} \end{bmatrix} \begin{bmatrix} pi_{fq} \\ pi_{kq} \\ pi_q \end{bmatrix} =$$

$$= \begin{bmatrix} \frac{\omega}{\omega_o}F_{41} & \frac{\omega}{\omega_o}F_{42} & \frac{\omega}{\omega_o}F_{43} & 0 & 0 & F_{46} \\ 0 & 0 & 0 & F_{54} & 0 & 0 \\ 0 & 0 & 0 & 0 & F_{65} & 0 \end{bmatrix} \begin{bmatrix} i_{fd} \\ i_{kd} \\ i_d \\ i_{fq} \\ i_{kq} \\ i_q \end{bmatrix} + \begin{bmatrix} V_q \\ V_{fq} \\ 0 \end{bmatrix} \qquad (2.35)$$

The A and E coefficients in 2.34 and 2.35 are given in Appendix I.

By separating derivatives in equations 2.34 and 2.35 and defining new coefficients (C, D whose values are given in Appendix I), the differential equations of the synchronous generator-infinite bus case, in matrix form are given as:

$$\begin{bmatrix} pi_{fd} \\ pi_{kd} \\ pi_d \\ pi_{fq} \\ pi_{kq} \\ pi_q \end{bmatrix} = \begin{bmatrix} A(1,1) & A(1,2) & A(1,3) & A(1,4)(1+n) \\ A(2,1) & A(2,2) & A(2,3) & A(2,4)(1+n) \\ A(3,1) & A(3,2) & A(3,3) & A(3,4)(1+n) \\ A(4,1)(1+n) & A(4,2)(1+n) & A(4,3)(1+n) & A(4,4) \\ A(5,1)(1+n) & A(5,2)(1+n) & A(5,3)(1+n) & A(5,4) \\ A(6,1)(1+n) & A(6,2)(1+n) & A(6,3)(1+n) & A(6,4) \end{bmatrix}$$

$$\begin{matrix} A(1,5)(1+n) & A(1,6)(1+n) \\ A(2,5)(1+n) & A(2,6)(1+n) \\ A(3,5)(1+n) & A(3,6)(1+n) \\ A(4,5) & A(4,6) \\ A(5,5) & A(5,6) \\ A(6,5) & A(6,6) \end{matrix}\Bigg] \begin{bmatrix} i_{fd} \\ i_{kd} \\ i_d \\ i_{fq} \\ i_{kq} \\ i_q \end{bmatrix} +$$

$$\begin{bmatrix} C_{11} & C_{12} & C_{13} & & & \\ C_{21} & C_{22} & C_{23} & & & \\ C_{31} & C_{32} & C_{33} & & & \\ & & & D_{11} & D_{12} & D_{13} \\ & & & D_{21} & D_{22} & D_{23} \\ & & & D_{31} & D_{32} & D_{33} \end{bmatrix} \begin{bmatrix} V_d \\ V_{fd} \\ 0 \\ V_q \\ V_{fq} \\ 0 \end{bmatrix} \qquad (2.36)$$

where $n = \omega_o p\delta$ and A(I,J) coefficients are given in Appendix I. The nonlinear system 2.36 will be used throughout this study.

2.5 Operating Point Calculation

An operating point is determined by P_o, Q_o and δ_o. The values of V_{fd}, V_{fq}, i_{fd} and i_{fq} have to be found and can be calculated using matrix form 2.36. Putting all derivatives equal to zero and rearranging i_d, i_q and V_{fd}, V_{fq} the set of linear algebraic equations 2.37 is to be solved.

$$\begin{bmatrix} A(1,1) & A(1,2) & C_{12} & A(1,4) & A(1,5) & 0 \\ A(2,1) & A(2,2) & C_{22} & A(2,4) & A(2,5) & 0 \\ A(3,1) & A(3,2) & C_{32} & A(3,4) & A(3,5) & 0 \\ A(4,1) & A(4,2) & 0 & A(4,4) & A(4,5) & D_{12} \\ A(5,1) & A(5,2) & 0 & A(5,4) & A(5,5) & D_{22} \\ A(6,1) & A(6,2) & 0 & A(6,4) & A(6,5) & D_{32} \end{bmatrix} \begin{bmatrix} i_{fd} \\ i_{kd} \\ i_d \\ i_{fq} \\ i_{kq} \\ i_q \end{bmatrix} =$$

$$\begin{bmatrix} C_{11}V_d & -A(1,3)i_d & 0 & 0 & -A(1,6)i_q & 0 \\ C_{21}V_d & -A(2,3)i_d & 0 & 0 & -A(2,6)i_q & 0 \\ C_{31}V_d & -A(3,3)i_d & 0 & 0 & -A(3,6)i_q & 0 \\ 0 & -A(4,3)i_d & 0 & -D_{11}V_q & -A(4,6)i_q & 0 \\ 0 & -A(5,3)i_d & 0 & -D_{21}V_q & -A(5,6)i_q & 0 \\ 0 & -A(6,3)i_d & 0 & -D_{31}V_q & -A(6,6)i_q & 0 \end{bmatrix} \qquad (2.37)$$

CHAPTER 3

CLOSED LOOP SUB-OPTIMAL CONTROL OF A D.W.R. SYNCHRONOUS GENERATOR

3.1 General

Modern optimal control theory has been increasingly applied, because a growing concern for the economic operation of complex industrial and manufacturing processes requires a new way of planning and design. Because of computer availability, optimal theory as an area of mathematics has made a significant impact on the design and operation of systems, small and large. The usefulness of optimization theory is dependent upon the ability to obtain numerical solutions. Having numerical solutions we can test the validity of conventional systems by comparison. Such a comparison must consider other factors such as the availability and cost of components for constructing the optimal control plus the cost and time required by the digital computer.

3.2 Statement of the Problem

For the fastest transient removal in the given system determine the controls V_{fd} and V_{fq} which force any given initial state $(\zeta_1, \zeta_2, \zeta_3)$ to the stable equilibrium point in minimum time. For most of this section the time

delays of the regulators on the direct and quadrature axes are neglected. The magnitude of the field voltages are bounded and are piecewise constant function of time (bang-bang control). Their signs are determined by the time optimal scheme. Now, the new control policy can be interpreted as in Fig. 3.1

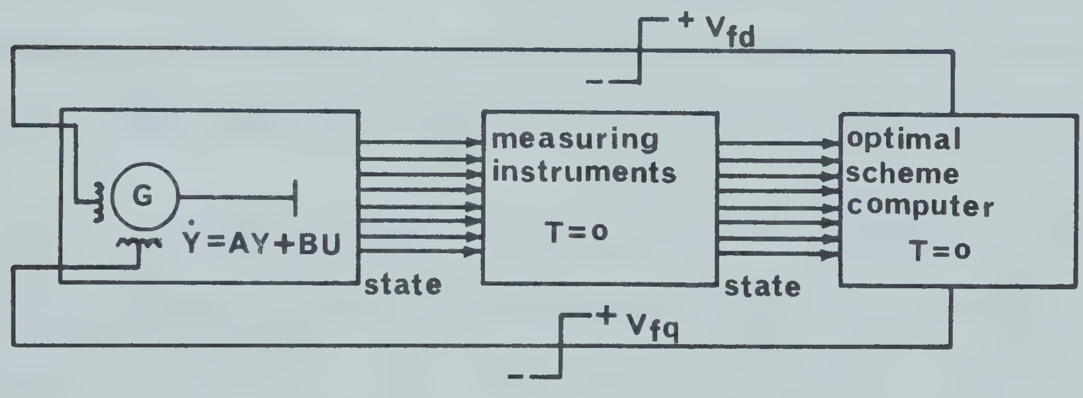

Fig. 3.1

The problem can be restated as:

Determine the control V_{fd} and V_{fq}, subject to constraints $|V_{fd}| \leq 5$, $|V_{fq}| \leq 5$, such that any initial state of the system $\mathring{Y} = YA + BU$ is transferred to a state determined by

$$\delta(t_f) = 0$$

$$\mathring{\delta}(t_f) = 0$$

$$0 \leq \delta \leq \pi/2 \tag{3.1}$$

in a minimum time.

3.3 Optimal Strategy

The swing equation of synchronous generator-infinite bus case is given by

$$T_j \, p^2\delta + K_d \, p\delta = T_t - T_{el} \tag{3.2}$$

Differentiating the above expression yields

$$T_j \, p^3\delta + K_d \, p^2\delta = -pT_{el} \tag{3.3}$$

The turbine torque is considered constant in this analysis. The first derivative of $T_{el} = \psi_d \, i_q - \psi_q \, i_d$ is

$$pT_{el} = i_q \, p\psi_d + \psi_d \, pi_q - i_d \, p\psi_q - \psi_q \, pi_d \tag{3.4}$$

where all the above derivatives are defined in section 2.1.

Substituting expressions for armature fluxes and eliminating a few terms because we are dealing with a nonsalient generator (i.e., $x_d = x_q$) equation 3.4 becomes

$$\begin{aligned} pT_{el} = {} & x_{afd} \, i_q \, pi_{fd} + x_{akd} \, i_q \, pi_{kd} + \\ & x_{ffd} \, i_{fd} \, p_{iq} + x_{fkd} \, i_{kd} \, pi_q \\ & -x_{afq} \, i_d \, pi_{fq} - x_{akq} \, i_d \, pi_{kq} \\ & -x_{afq} \, i_{fq} \, pi_d - x_{akq} \, i_{kq} \, pi_d \end{aligned} \tag{3.5}$$

Further elimination of derivatives in expression 3.5 and grouping of terms is given in Appendix II. Using the results from Appendix II expression 3.3 is

$$T_j p^3 \delta + K_d p^2 \delta = -(E + V_{fd}\ BN + V_{fq}\ BK) \qquad (3.6)$$

or

$$p^3 \delta + \frac{\omega_o K_d}{6}\ p^2 \delta = -\ \frac{\omega_o}{6}(E + V_{fd}\ BN + V_{fq}\ BK) \qquad (3.7)$$

Further simplification gives

$$p^3 \delta + K\ p^2 \delta = T \qquad (3.8)$$

where $K = \dfrac{\omega_o\ K_d}{6}$ is a positive constant and T behaves as a controlling function.

Defining the variables $Y_1(t)$, $Y_2(t)$, $Y_3(t)$

$$Y_1(t) = \delta$$

$$Y_2(t) = p\delta$$

$$Y_3(t) = p^2 \delta \qquad (3.9)$$

The state space equations of 3.8 are

$$\begin{bmatrix} pY_1(t) \\ pY_2(t) \\ pY_3(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -K \end{bmatrix} \begin{bmatrix} Y_1(t) \\ Y_2(t) \\ Y_3(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ T(t) \end{bmatrix} \qquad (3.10)$$

or in a vector-matrix form

$$\mathring{Y}(t) = A\ Y(t) + T \tag{3.11}$$

The matrix A can be transferred to its Jordan canonical form because two of three eigenvalues are the same. The transformation matrix P is given such the relation $J(A) = P^{-1}\ AP$ is satisfied, i.e.

$$P = \begin{bmatrix} 1 & 0 & \frac{1}{K^2} \\ 0 & 1 & -\frac{1}{K} \\ 0 & 0 & 1 \end{bmatrix} \tag{3.12}$$

The vector Y(t) can be transformed into vector Z(t) directly by the following

$$Z(t) = P^{-1}Y(t) \tag{3.13}$$

Differentiating this equation yields

$$p\ Z(t) = P^{-1}\ pY(t) \tag{3.14}$$

and then eliminating vector Y(t)

$$p\ Z(t) = P^{-1}\ A\ P\ Z(t) + P^{-1}\ T(t) \tag{3.15}$$

or

$$p\ Z(t) = J(A)\ Z(t) + P^{-1}\ T(t) \tag{3.16}$$

results in the system

$$p\,Z_1(t) = Z_2(t) - \frac{1}{K^2}\,T(t)$$

$$p\,Z_2(t) = \frac{1}{K}\,T(t)$$

$$p\,Z_3(t) = -K - Z_3(t) + T(t) \qquad (3.17)$$

For simplicity, new state variables $x_1(t)$, $x_2(t)$, $x_3(t)$ can be defined by setting

$$x_1(t) = K^3\,Z_1(t)$$

$$x_2(t) = K^2\,Z_2(t)$$

$$x_3(t) = K\,Z_3(t) \qquad (3.18)$$

After this linear transformation the system of differential equations in the variable x are

$$px_1(t) = K\;x_2(t) - K\,T(t)$$

$$px_2(t) = K\,T(t)$$

$$px_3(t) = -K\,x_3(t) + K\,T(t) \qquad (3.19)$$

Now, having system 3.19 our problem can be restated. For a given system 3.19 find the admissible control that forces the system of Fig. 3.1 from any initial state $(\zeta_1, \zeta_2, \zeta_3)$ to the stable operating point in minimum time. The general expression for the Hamiltonian H for the minimum-time control of the system 3.19 is

$$H = 1 + \sum_{i=1}^{3} f_i[x(t),t]\ \lambda_i(t) + \sum_{j=1}^{3} T_j(t)\{\Sigma b_{ij}[x(t),t]\lambda_i(t)\} \tag{3.20}$$

Equation 3.20 applied to system 3.19 gives

$$H = 1 + K\ x_2\ \lambda_1 - K\ x_3\ \lambda_3 + K\ T(t)(-\lambda_1(t) + \lambda_2(t) + \lambda_3(t)) \tag{3.21}$$

The H minimal control, i.e., the control which minimizes the Hamiltonian, is given by

$$\operatorname{sgn} T(t) = -\operatorname{sgn}\{-\lambda_1(t) + \lambda_2(t) + \lambda_3(t)\} \tag{3.22}$$

where the control variable is constrained by the inequality

$$T_{min} \le T(t) \le T_{max}$$

The costate variables $\lambda_1(t)$, $\lambda_2(t)$ and $\lambda_3(t)$ satisfy the equations

$$\begin{aligned} p\lambda_1(t) &= -\frac{\partial H}{\partial x_1} = 0 \\ p\lambda_2(t) &= -\frac{\partial H}{\partial x_2} = -K\ _1(t)\ \lambda_1(t) \\ p\lambda_3(t) &= -\frac{\partial H}{\partial x_3} = K\ _3(t)\ \lambda_3(t) \end{aligned} \tag{3.23}$$

Let π_1, π_2 and π_3 be the initial values of the costate functions, then the solutions of the system 3.23 are

$$\lambda_1(t) = \pi_1$$

$$\lambda_2(t) = \pi_1 - K\ \pi_2 t$$

$$\lambda_3(t) = \pi_3\ e^{Kt}$$

The function

$$f(\lambda) = -\lambda_1(t) + \lambda_2(t) + \lambda_3(t) = -\pi_1 + \pi_2 - K\pi_1 t + \pi_3 e^{-Kt}$$

is a linear combination of constants, linear function and one exponential function. This kind of function can have at most two zeros as shown in Fig. 3.2.

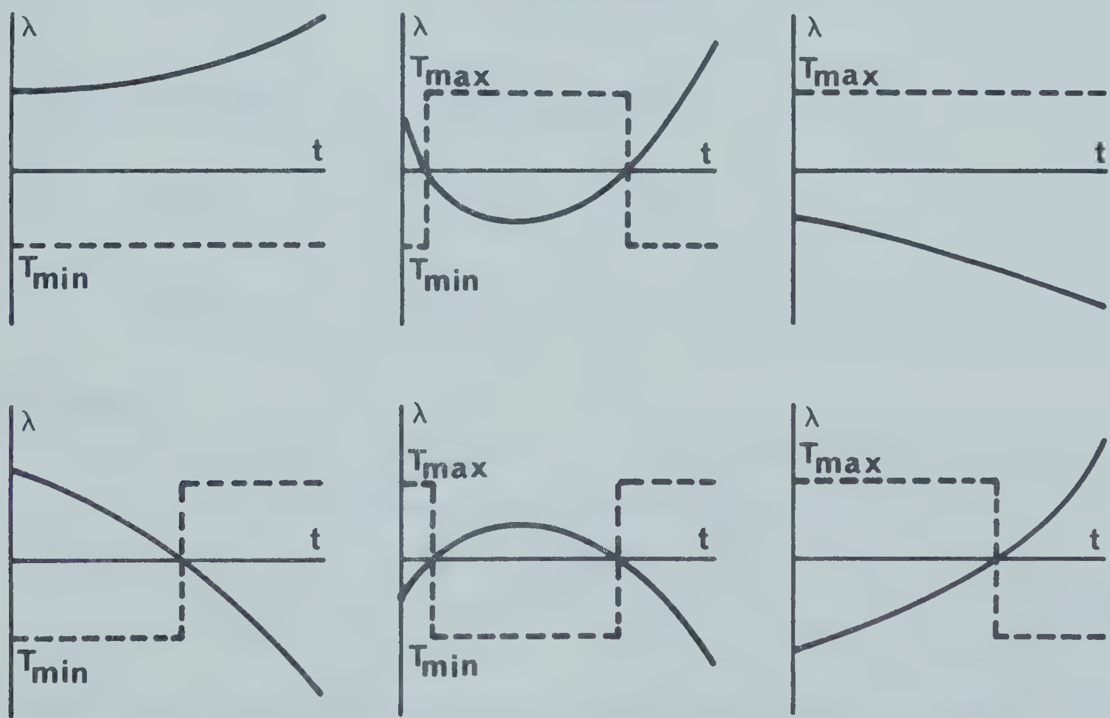

Fig. 3.2

Since, over a finite interval of time, the time optimal control is constant, $T = T_{max}$ or $T = T_{min}$, the equa-

tion 3.19 can be solved using $T(t) = \text{const}$, with the initial conditions $x_1(0) = \zeta_1$, $x_2(0) = \zeta_2$, $x_3(0) = \zeta_3$

$$x_1(t) = \zeta_1 + \zeta_2 K t + \tfrac{1}{2} T K^2 t^2 - T K t$$

$$x_2(t) = \zeta_2 + T K t$$

$$x_3(t) = (\zeta_3 - T)e^{-Kt} + T \tag{3.25}$$

Eliminating the time t from the second equation of the set 3.25

$$t = \frac{x_2(t) - \zeta_2}{T K}$$

and then substituting in equations

$$x_1(t) = \zeta_1 + \zeta_2 K t + \tfrac{1}{2} T K^2 t^2 - T K t$$

$$x_3(t) = (\zeta_3 - T)e^{-Kt} + T$$

we have

$$x_1 = \tfrac{1}{2} \frac{x_2^{\,2}}{T} - x_2 + \zeta_1 - \tfrac{1}{2} \frac{\zeta_2^{\,2}}{T} + \zeta_2$$

$$x_3 = (\zeta_3 - T)e^{-\frac{x_2 - \zeta_2}{T}} + T \tag{3.26}$$

These two equations determine the trajectory in the three-dimensional state space (with the initial state ζ_1, ζ_2, ζ_3).

The equation

$$x_1 = \tfrac{1}{2} \frac{x_2^{\,2}}{T} - x_2 + \zeta_1 - \tfrac{1}{2} \frac{\zeta_2^{\,2}}{T} + \zeta_3$$

is the equation of the trajectories which are parabolic. The family of these curves is shown in Fig. 3.3

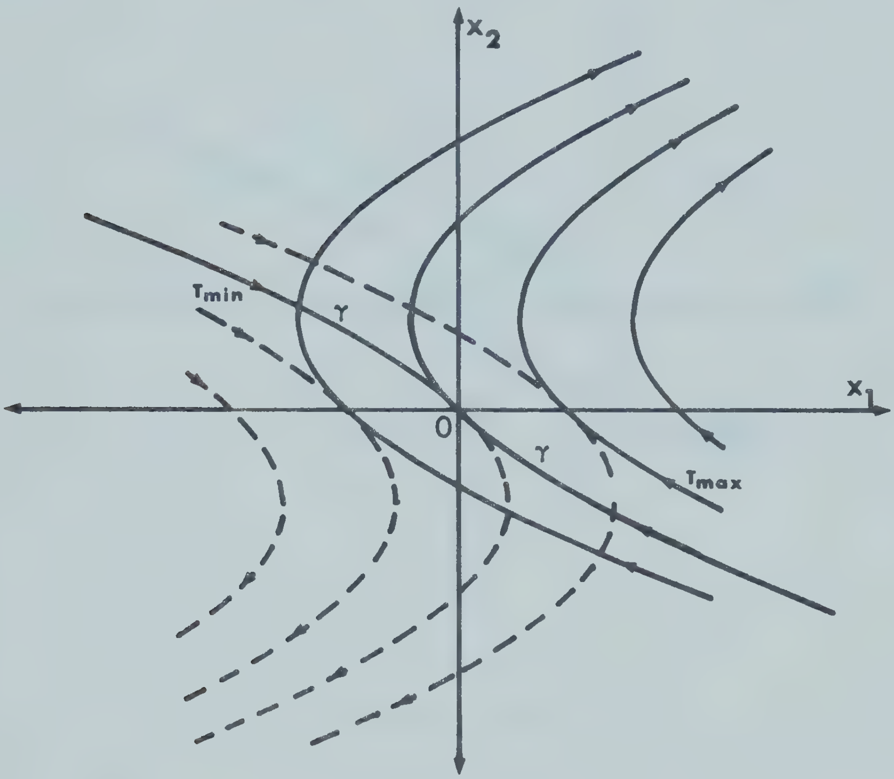

Fig. 3.3

For some cases our objective is to drive any initial state to the origin of the state plane. The parts of the above two curves combined into the γ curve are the locuses of all points (x_1, x_2) which can be forced to the origin or by the control T_{max} or by the control T_{min}. Let us now consider an initial state (ζ_1, ζ_2) which does not belong to the γ curve (Fig. 3.4).

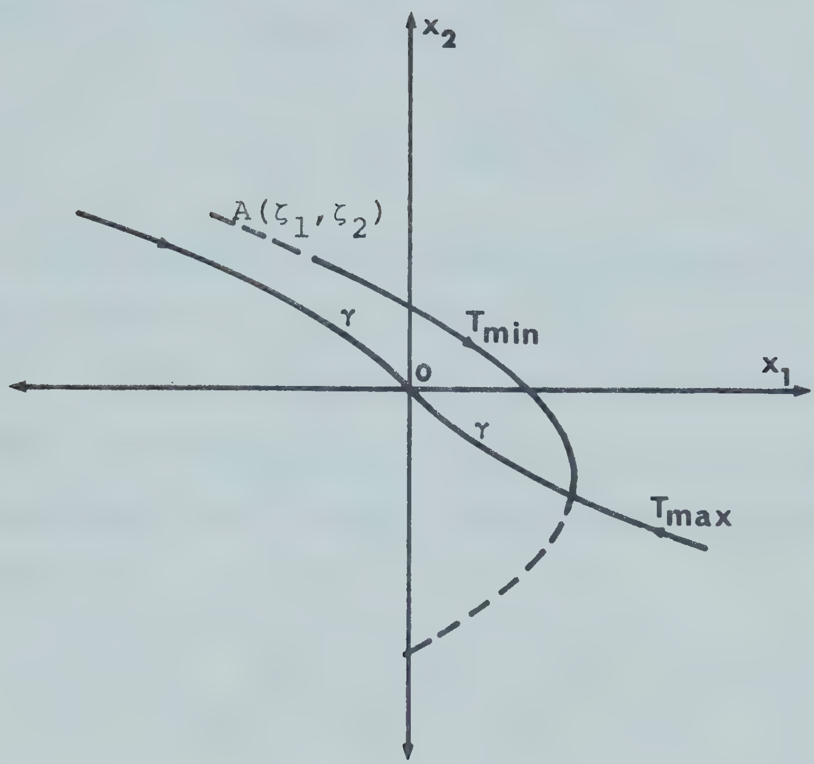

Fig. 3.4

In this case a control T_{min} must be applied to bring the system to the γ switch curve (use of the other control T_{max} will never reach the γ curve). At that point the transition occurs from the control T_{min} to the control T_{max} and the system reaches the origin along the γ switch curve (with the control T_{max}).

The γ curve is given by the equation

$$x_1 = \frac{1}{2T} x_2^{\,2} - x_2 \tag{3.27}$$

where $T = T_{max}$ if $x_2>0$ and $T = T_{min}$ if $x_2<0$. The second equation of the system (3.26)

$$x_3 = (\zeta_3 - T)e^{\frac{-x_2-\zeta_2}{T}} + T \tag{3.28}$$

gives the trajectories in the x_2, x_3 plane with various initial states (ζ_2, ζ_3). Fig. 3.5.

Let us consider the two trajectories which pass through the origin in the x_2, x_3 plane. By separating the variables and the initial coordinates in 3.28 the result yields

$$x_3 e^{\frac{x_2}{T}} - Te^{\frac{x_2}{T}} = \zeta_3 e^{\frac{\zeta_2}{T}} - Te^{\frac{\zeta_2}{T}} \tag{3.29}$$

The two curves passing through the origin of the x_2, x_3 plane have to have initial conditions such that the right hand side of equation 3.29 is equal to -T, i.e.

$$x_3 e^{\frac{x_2}{T}} - Te^{\frac{x_2}{T}} = -T \tag{3.30}$$

or

$$x_3 = T - Te^{\frac{-x_2}{T}} \tag{3.31}$$

where $T=T_{min}$ if $x_2>0$ and $T=T_{max}$ if $x_2<0$ (Fig. 3.5)

For the disturbances when the rotor angle has a maximum excursion less than $\frac{\pi}{2\omega_o}$ the control scheme can be decided from the x_2, x_3 plane. By reaching the origin

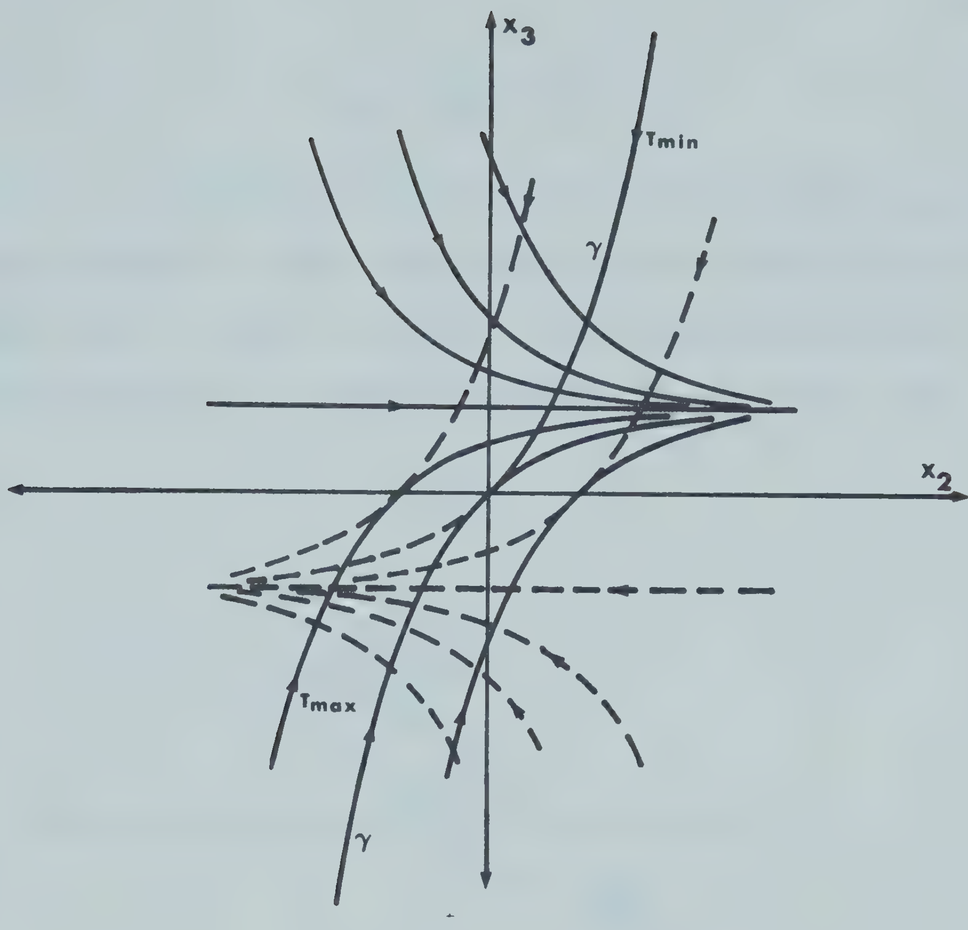

Fig. 3.5

of the x_2, x_3 plane (i.e. $x_2 = 0$, $x_3 = 0$) two of three conditions for equilibrium are satisfied, because $x_2 = 0$ and $x_3 = 0$ implies $p\delta = 0$ and $p^2\delta = 0$. The third one $(0 \le \delta \le \frac{\pi}{2\omega_o})$ is already satisfied. For cases in which the rotor angle is larger than $\frac{\pi}{2\omega_o}$ the control scheme has to be decided from the combination of the x_1, x_2 and x_2, x_3 planes, i.e., from the switching function in space x_1, x_2, x_3. The projection of the switch curve γ in the x_1, x_2 plane is given by 3.26.

$$x_{11} = \frac{1}{2T} x_{21}^2 - x_{21} \qquad (3.32)$$

Let us now consider an initial state (x_1, x_2, x_3) which belongs to the trajectory in the space which intersects the γ curve, and such that a system staying on that trajectory can be forced to reach the γ curve Fig. 3.6.

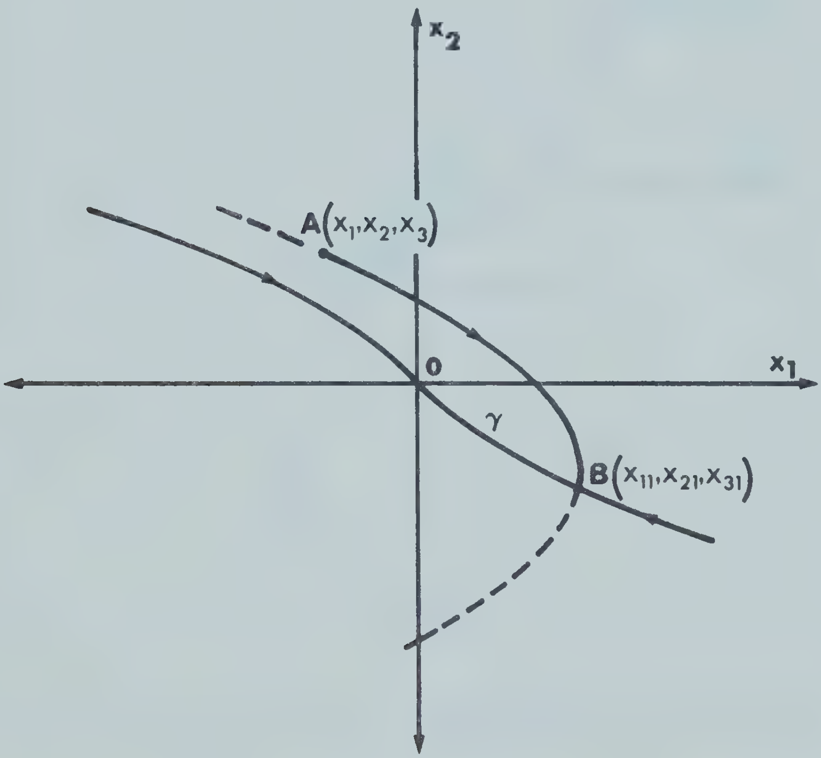

Fig. 3.6

The equation of the path AB is

$$x_{12} = -\frac{1}{T} x_{22}^2 - x_{22} + x_1 + \frac{1}{2T} x_2^2 + x_2 \qquad (3.33)$$

An analysis similar to that used before, in the x_2, x_3 plane with the equation of γ curve

$$x_{31} = T - Te^{\frac{-x_{21}}{T}}$$

leads to an equation of the path CD (Fig. 3.7)

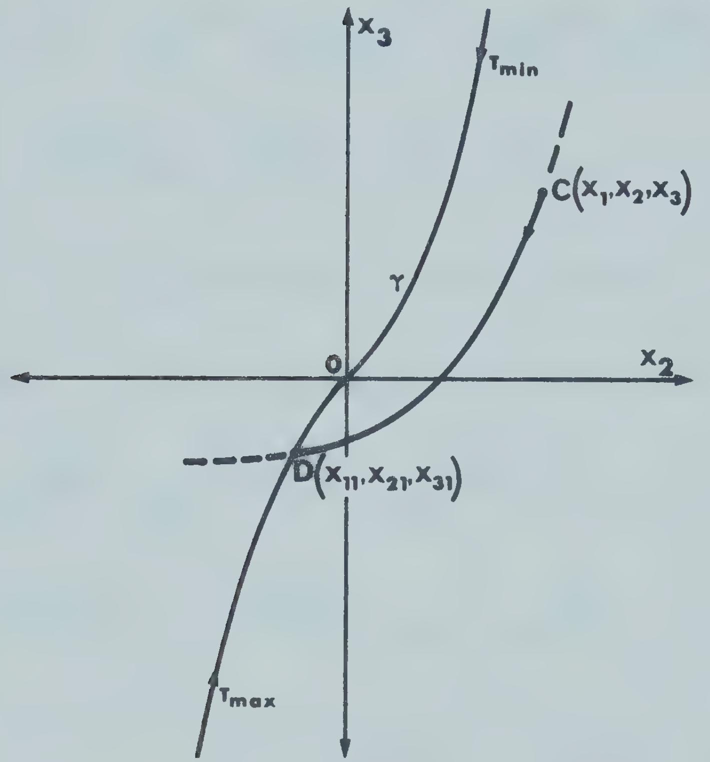

Fig. 3.7

$$x_{32} = (x_3 + T)e^{\frac{1}{T}(x_{22} - x_2)} - T$$

Now, we have four equations and two unknowns

$$x_{12} = -\frac{1}{2T} x_{22}^2 - x_{22} + x_1 - \frac{1}{2T} x_2^2 + x_2 \tag{3.34}$$

$$x_{32} = (x_3 + T)e^{\frac{1}{T}(x_{22} - x_2)} - T \tag{3.35}$$

$$x_{12} = \frac{1}{2T} x_{22}^2 - x_{22} \tag{3.36}$$

$$x_{32} = T - T e^{-\frac{x_{22}}{T}} \tag{3.37}$$

Eliminate x_{12}, x_{22}, x_{32} from the set of equations 3.34-3.37 by first finding x_{22} from equations 3.34 and 3.37

$$\frac{1}{2T} x_{22}^{2} - x_{22} = -\frac{1}{2T} x_{22}^{2} - x_{22} + x_1 \frac{1}{2T} x_2^{2} + x_2$$

$$\frac{1}{T} x_{22}^{2} = \frac{1}{2T} x_2^{2} + x_1 + x_2$$

$$x_{22} = \pm(\tfrac{1}{2}x_2^{2} + T(x_1 + x_2))^{\frac{1}{2}} \tag{3.38}$$

Eliminate x_{32} from equations 3.35 and 3.37

$$T - Te^{-\frac{x_{22}}{T}} = x_3 e^{\frac{x_{22}-x_2}{T}} + Te^{\frac{x_{22}-x_2}{T}} - T$$

$$x_3 e^{\frac{x_{22}-x_2}{T}} = -Te^{\frac{x_{22}-x_2}{T}} + 2T - Te^{-\frac{x_{22}}{T}} \tag{3.39}$$

or

$$x_3 = -T + 2Te^{-\frac{x_{22}-x_2}{T}} - Te^{-\frac{x_{22}-x_2}{T}} e^{-\frac{x_{22}}{T}} \tag{3.40}$$

Substituting equation 3.37 into equation 3.39 gives

$$x_3 = -T + Te^{-\frac{1}{T}\{\pm[\frac{1}{2}x_2^{2} + T(x_1+x_2)]^{\frac{1}{2}} - x_2\}}$$
$$(2 - e^{-\frac{1}{T}\{\pm[\frac{1}{2}x_2^{2} + T(x_1+x_2)]^{\frac{1}{2}}\}}) \tag{3.41}$$

In equation 3.41 there is an ambiguity of sign. Going back to the equations 3.36 and 3.37, i.e., to the γ curves in the x_1,x_2; x_2,x_3 planes respectively the con-

trol T (T_{min} or T_{max}) is determined as

$$\operatorname{sgn} T = -\operatorname{sgn}\{x_{22}\} \tag{3.42}$$

or

$$x_{22} = -\operatorname{sgn}T\left[\frac{x_2^{\ 2}}{2} + T(x_1+x_2)\right]^{\frac{1}{2}} \tag{3.43}$$

So the final expression for the switching surface is

$$x_3 = -T + Te^{\left\{\frac{1}{|T|}[\frac{1}{2}x_2^{\ 2} + T(x_1+x_2)]^{\frac{1}{2}} + \frac{1}{T}x_2\right\}} \left(2 - e^{\frac{1}{|T|}[\frac{1}{2}x_2^{\ 2} + T(x_1+x_2)]^{\frac{1}{2}}}\right) \tag{3.44}$$

where $|T|$ is the absolute value of T, or

$$\Sigma = x_3 + T - T\exp\left(\frac{1}{|T|}[\tfrac{1}{2}x_2^{\ 2} + T(x_1+x_2)]^{\frac{1}{2}} + \frac{1}{T}x_2\right) \left(2 - \exp\left\{\frac{1}{|T|}[\tfrac{1}{2}x_2^{\ 2} + T(x_1+x_2)]^{\frac{1}{2}}\right\}\right) \tag{3.45}$$

The control $T = T_{min}$ or $T = T_{max}$ has to be decided such that the trajectory generated by one of them intersects the switching surface (Fig. 3.8). For a state above the switching surface the control is $T = T_{max}$, and for a state below the switching surface the control is $T = T_{min}$.

In the control scheme we first have to find the value of the angle δ. If the angle is out of the range $\pm \frac{\pi}{2\omega_o}$ the switching surface has to be used to bring the angle δ into the range $\pm \frac{\pi}{2\omega_o}$. After that the control

scheme from the plane x_2, x_3 is decided.

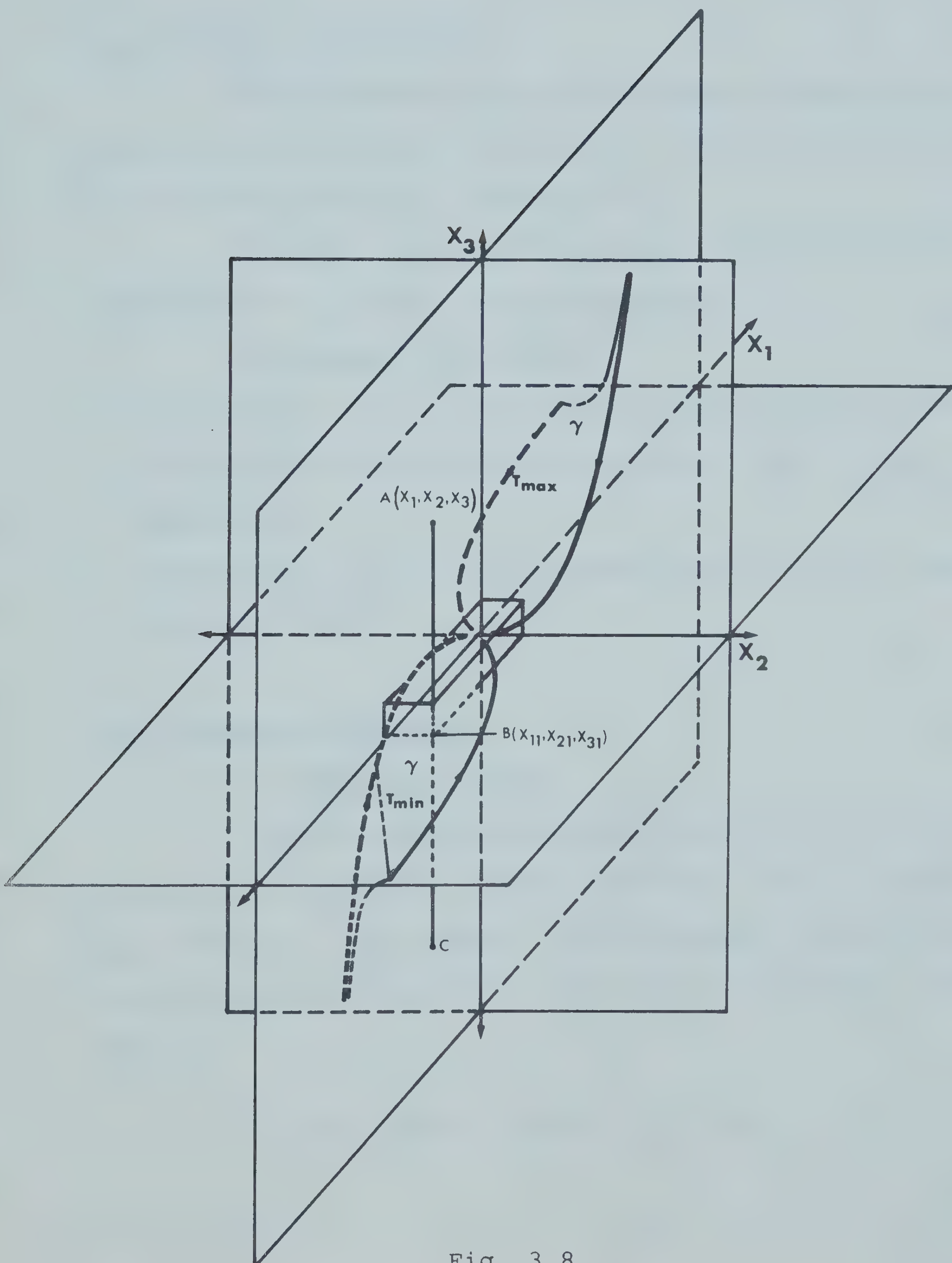

Fig. 3.8

CHAPTER 4

4.1 Method of Simulation

The equations were solved using a Runge-Kutta-Gill 4th order integration routine with a step size of 0.001 sec. on an IBM-360 model 67 (Appendix III).

Several forms of disturbance were assumed for computation of transient characteristics.

Disturbance considered:

a) Torque steps of 10, 30 and 50% of the system load for a single machine infinite bus system. Fig. 2.4 and Appendix I.

b) Torque pulse of 100% for the same system and the same operating points.

4.2 Excitation Controls

1. Bang-Bang Control

From the optimal trajectories and the switching function note that the control function T has two values. T_{max} and T_{min}. The optimal scheme requires that the value of T changes instantaneously from one to the other. T is given as

$$T = -\frac{\omega_o}{6}[E + V_{fd}\ BN + V_{fq}\ BK]$$

To change the value of T to either T_{min} or to the T_{max} the actual controls have to change from $V_{fd\ min}$, $V_{fq\ min}$ to $V_{fd\ max}$, $V_{fq\ max}$.

The field voltages are constrained in magnitude. Usually this constraint is ±5 units.

The angle time characteristics (Fig. 4.1) show responses of controlled and uncontrolled machine for a 10% torque step. For the uncontrolled case the system stabilizes but is poorly damped. For the machine (Fig. 4.1-b) with bang-bang control, the time required to reach a stable point is short and the response is heavily damped. The phase plane plot (Fig. 4.2) shows the nature of the forced and free damped oscillatory systems. For the forced system (Fig. 4.2) the rotor velocity is brought near a stable equilibrium point in a fairly short period of time but exhibits forced oscillations characterised by slow variations of speed due to the large rotor inertia.

The responses for more severe disturbances of 30% and 50% torque steps are shown in Fig. 4.3 and Fig. 4.4. As can be expected the final angle becomes bigger and the time required to reach a stable point is increased. Angle time characteristics from Fig. 4.3 and Fig. 4.4 are slightly oscillatory particularly in the initial period. To analyse this we have to go back to the switching trajectory. Starting from any initial state we want the forced trajec-

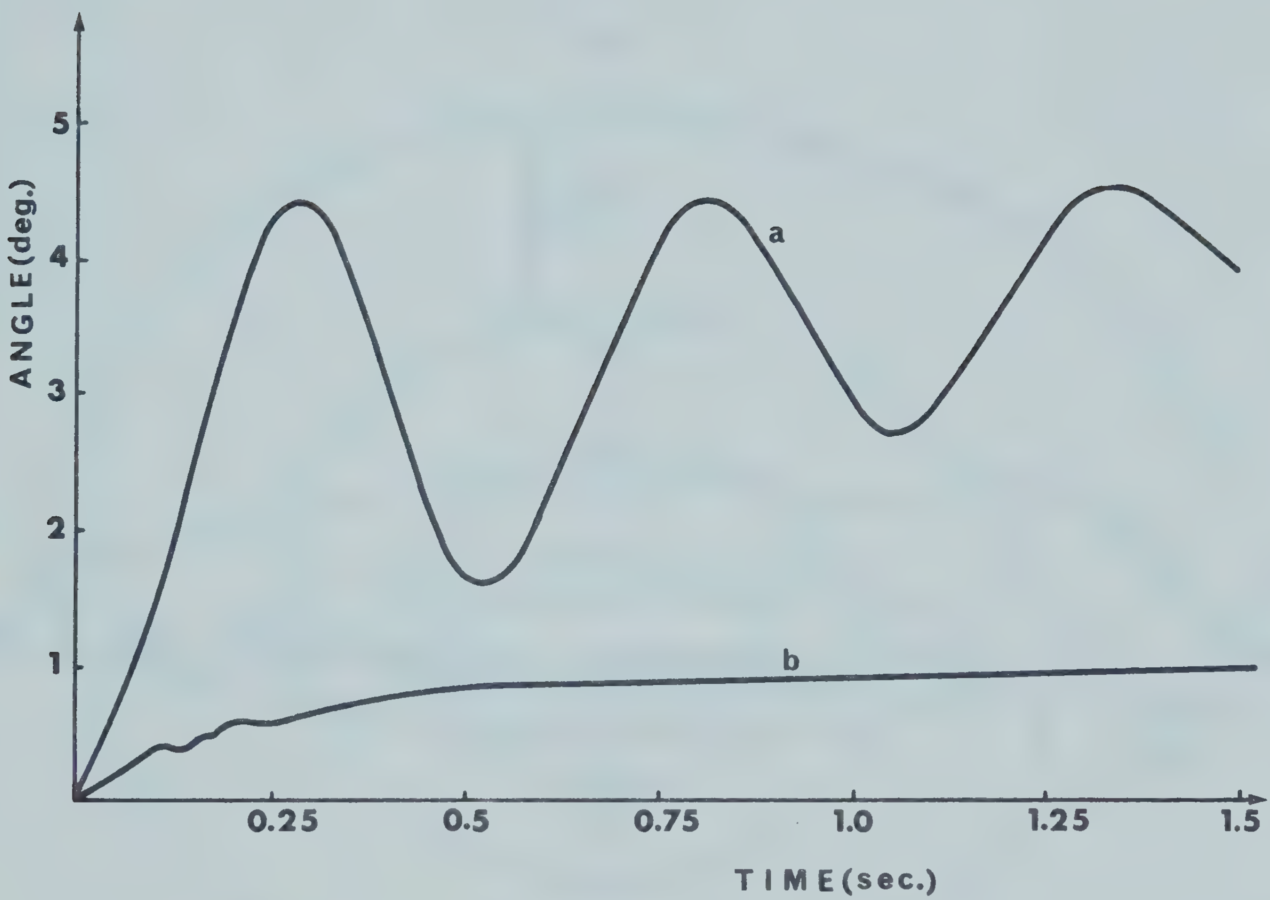

Fig. 4.1

a) unregulated machine

b) regulated machine (bang-bang control)

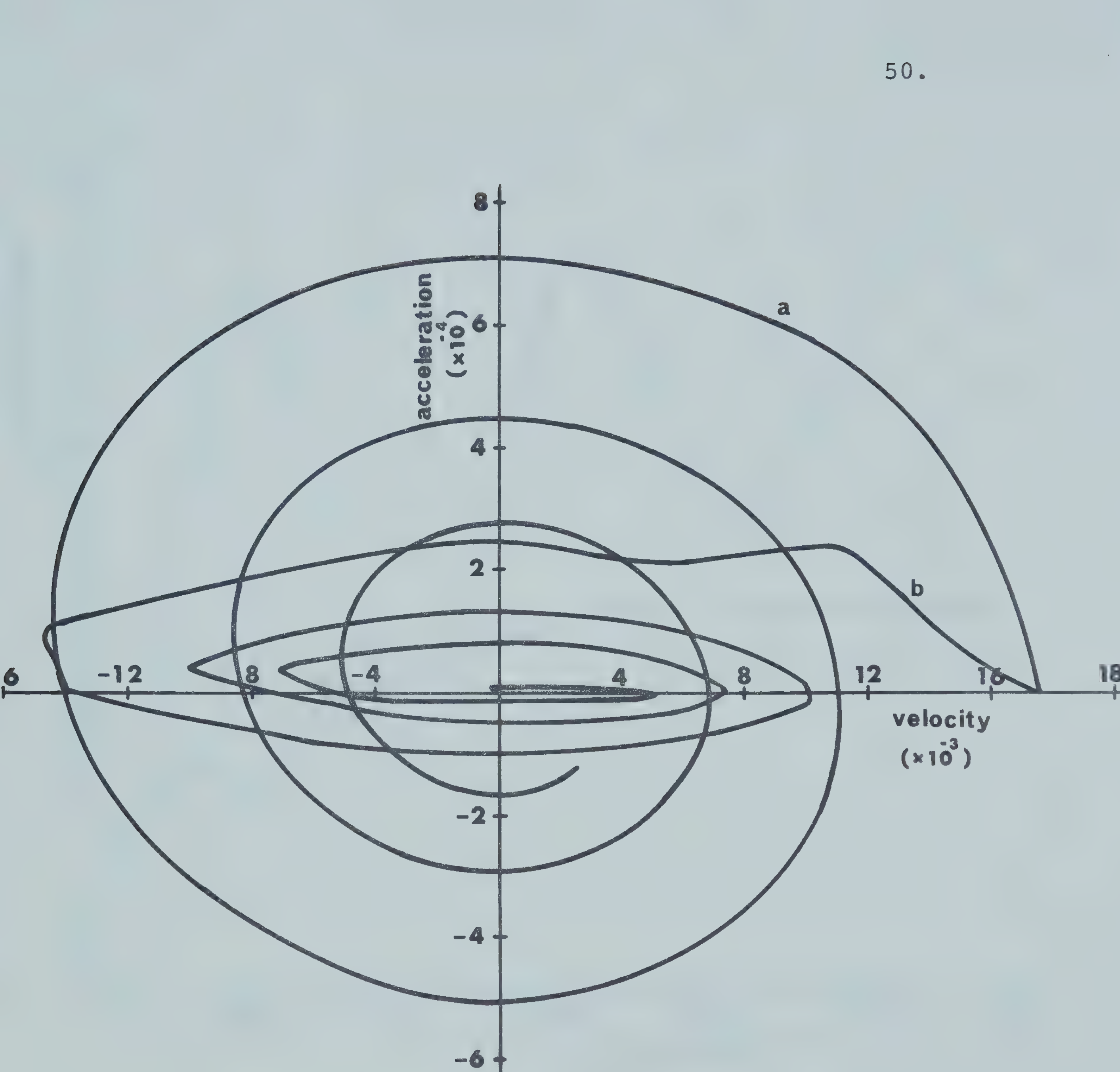

Fig. 4.2

Velocity vs acceleration plot corresponding to Fig. 4.1

a) unregulated machine 10% torque step

b) regulated machine (bang-bang control)

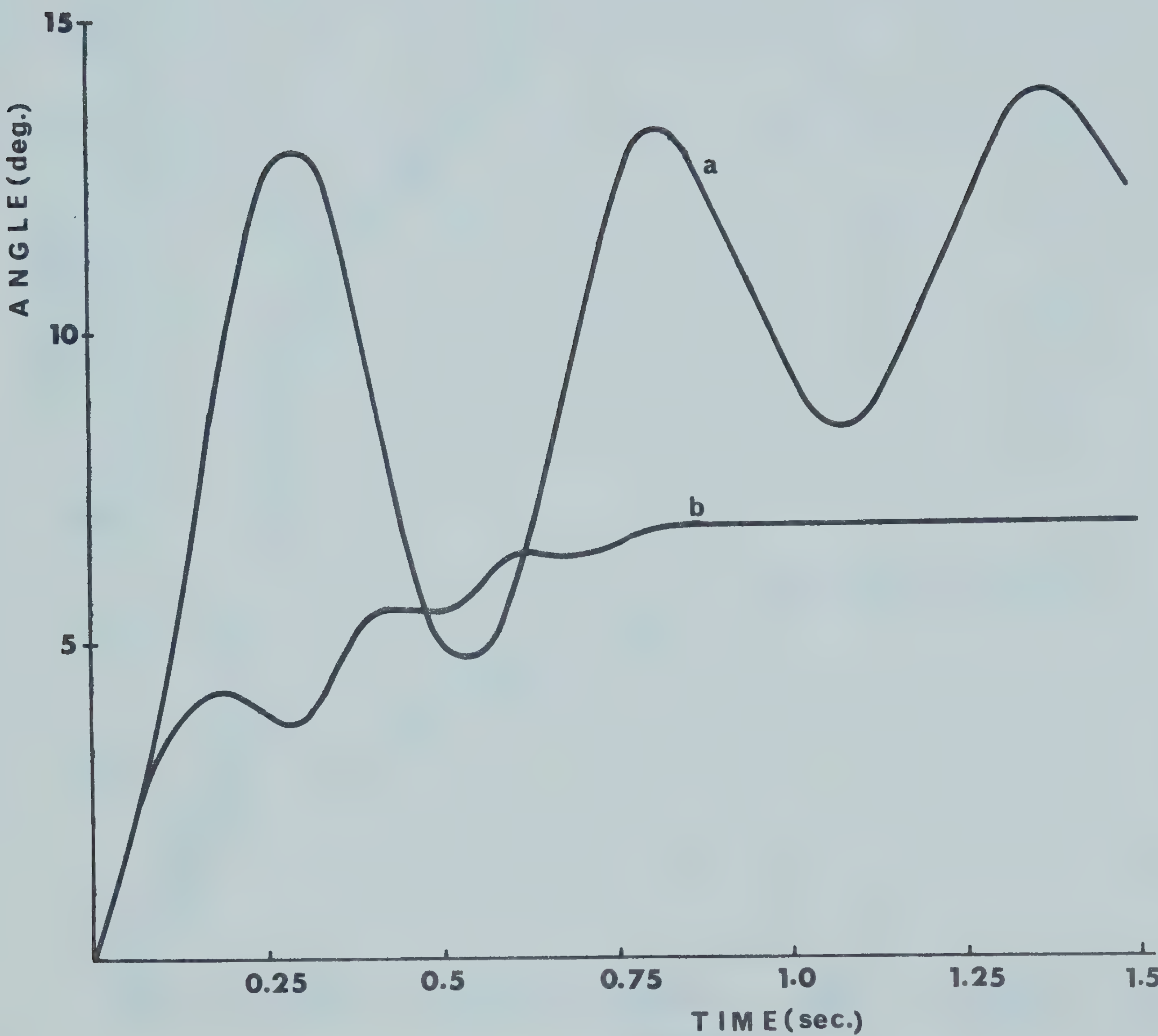

Fig. 4.3

Angle time characteristics for 30% torque step

a) unregulated machine

b) regulated machine

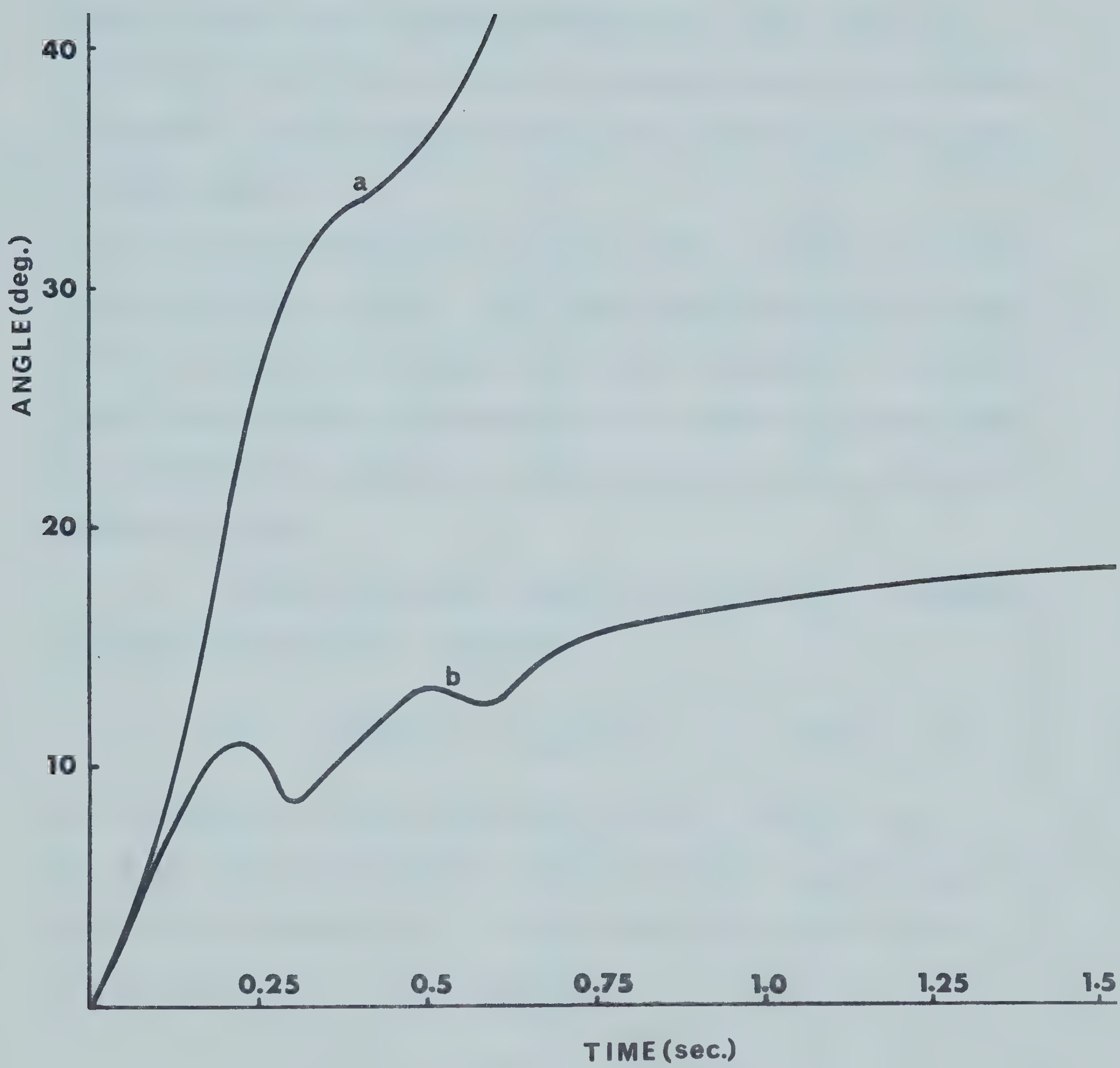

Fig. 4.4

Angle time characteristics for 50% torque step

a) unregulated machine

b) regulated machine (bang-bang control)

tory to reach the switching trajectory, and after the transition from one control to another occurs, the forced trajectory should continue along the switching trajectory to the origin.

Due to system imperfections the forced trajectory parallels the switch curve. For a big disturbance and strong control the forced trajectory tends to recross the switch curve after having switched once and control action steps back and forth causing the trajectory to zigzag along the switching curve.

A more important cause of oscillatory responses is due to the control function.

$$T(t) = -\frac{\omega_o}{6}\left[E(t) + V_{fd}\ BN(t) + V_{fq}\ BK(t)\right]$$

is a function of the terms E(t), BN(t), BK(t) (i.e., a function of state) and the field voltages V_{fd} and V_{fq}. From this expression it is seen that the system can be controlled only if the relation

$$E(t) < (V_{fd}\ BN(t) + V_{fq}\ BK(t))$$

is satisfied. By checking the variation of E(t), BN(t) and BK(t) it was noted that there were uncontrollable initial periods of short duration.

Fig. 4.5 shows the comparison between the variation of field voltages and field currents. The stepwise

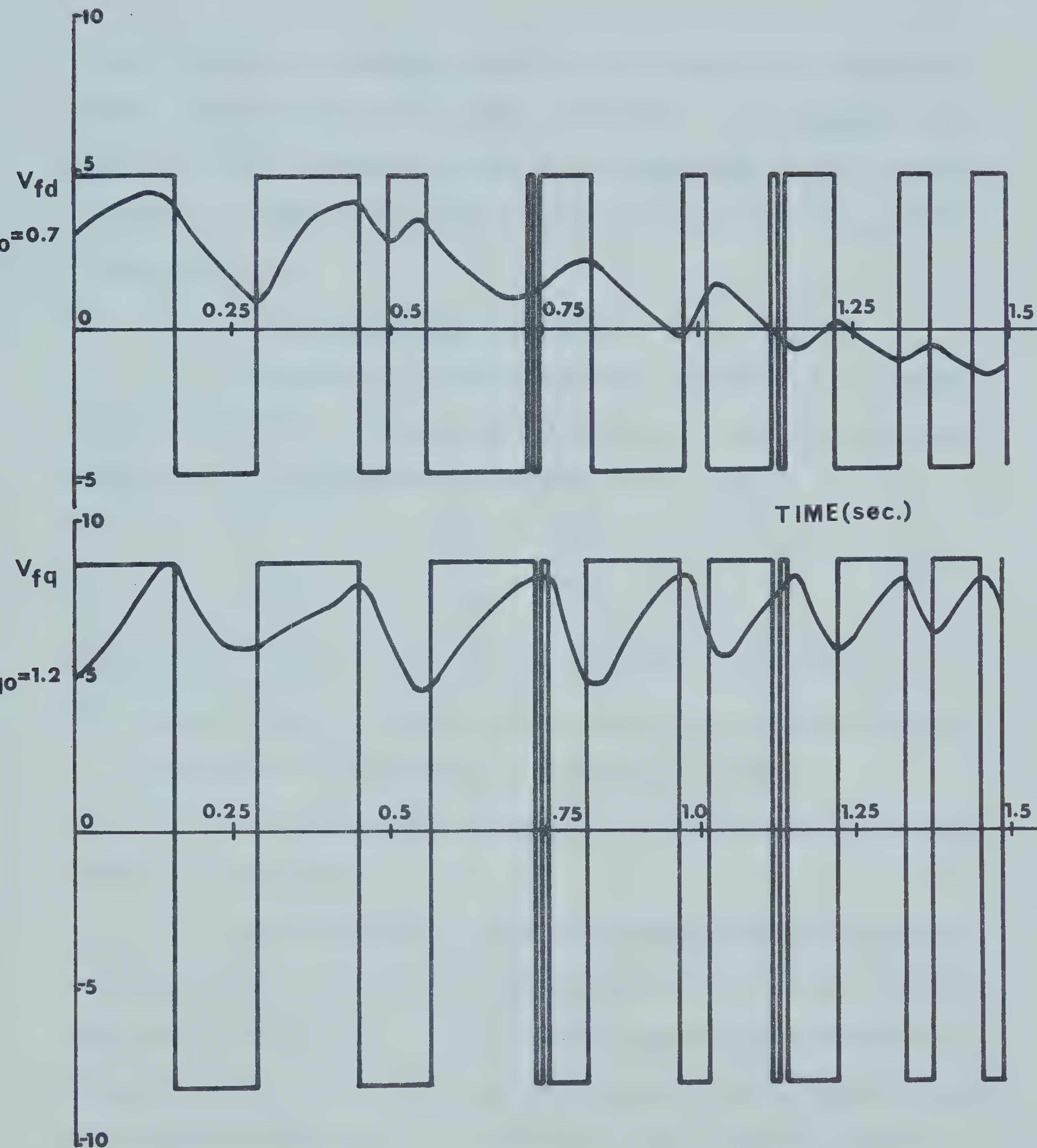

Fig. 4.5

Field voltages and field currents corresponding to

50% torque step

field voltage variation cannot be followed by field currents, because of field time constants. In contrast to a single field machine the field currents may reach negative values to produce increased synchronizing (or resynchronizing) torque.

2. Proportional Control

Instead of using bang-bang control, i.e., the values of field voltages decided by the sign of switching function, a proportional control may be used.

$$\begin{aligned} V_{fd} &= -K_1\Sigma \\ V_{fq} &= -K_2\Sigma \end{aligned} \qquad (4.2)$$

With constrains on V_{fd} and V_{fq} as defined in 3.2

The values of $K_{1,2}$ should be as large as possible for $K_{1,2} = \infty$, equations 4.2 reduce to a bang-bang control.

Unfortunately large K_1, K_2 may throw the system into a steady hunt.

The comparison between a bang-bang and proportional control for a 100% torque pulse of 3 cycles duration is given in Fig. 4.6. The angle time characteristics in Fig. 4.6 show that the bang-bang control has smaller first and second overshoots. The final rotor angle is also smaller than for the proportional control.

From Fig. 4.11 it can be seen that the bang-bang and proportional control deteriorate the terminal voltage,

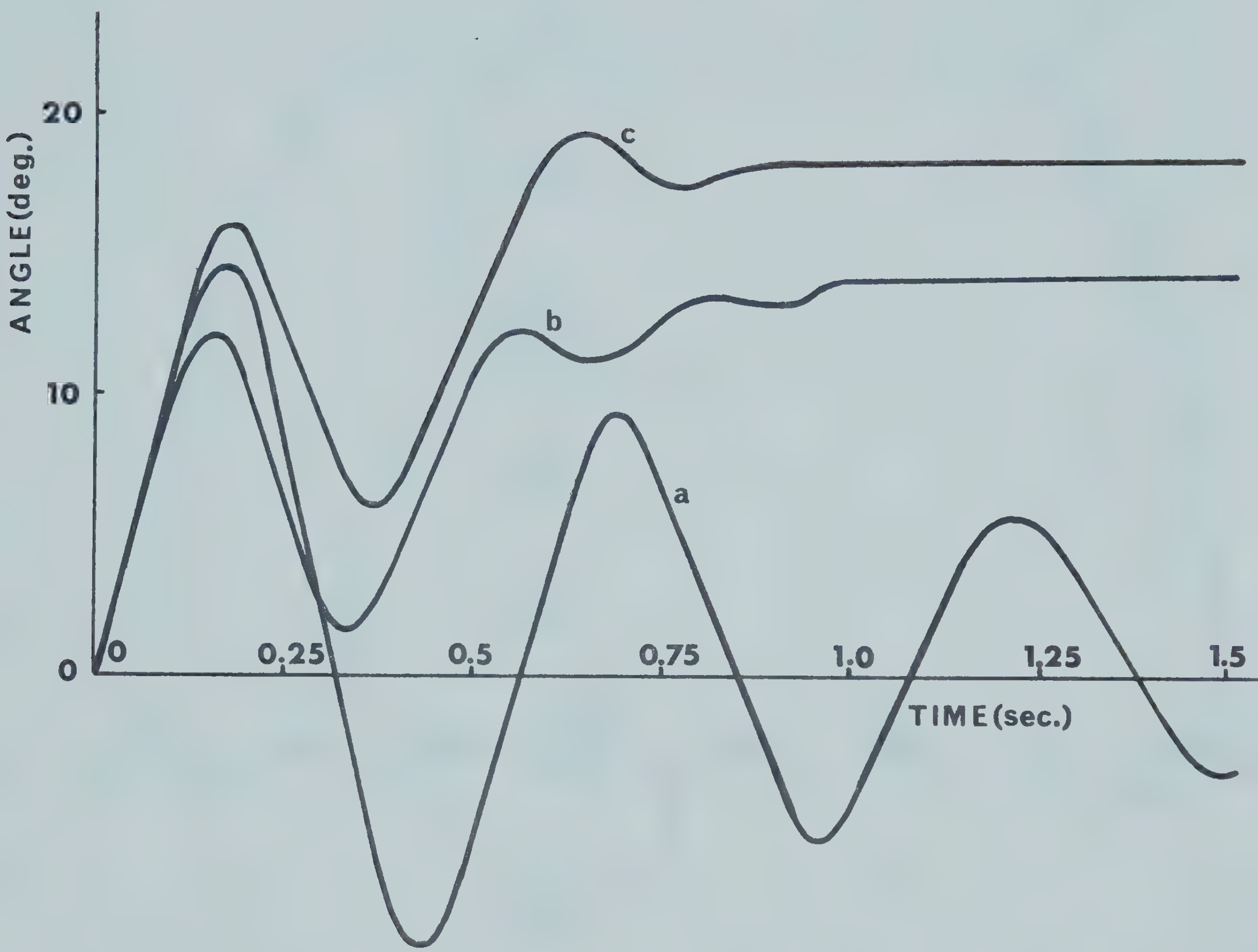

Fig. 4.6

Angle time characteristics for 100% torque pulse (3 cycles)

a) unregulated machine

b) regulated machine (bang-bang control)

c) regulated machine (proportional control)

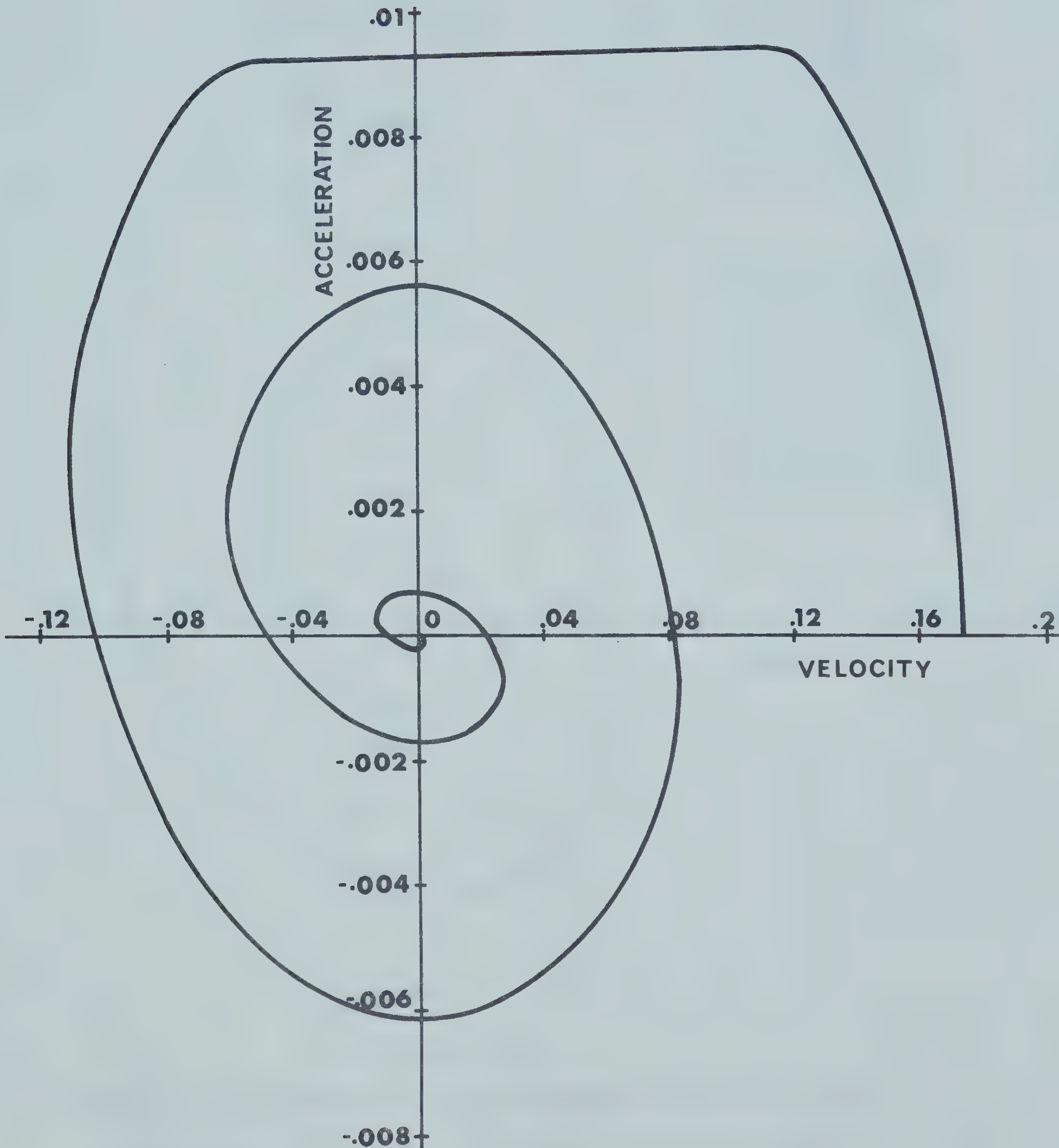

Fig. 4.7

Velocity vs acceleration plot corresponding to Fig. 4.6

100% torque pulse (case b, bang-bang control)

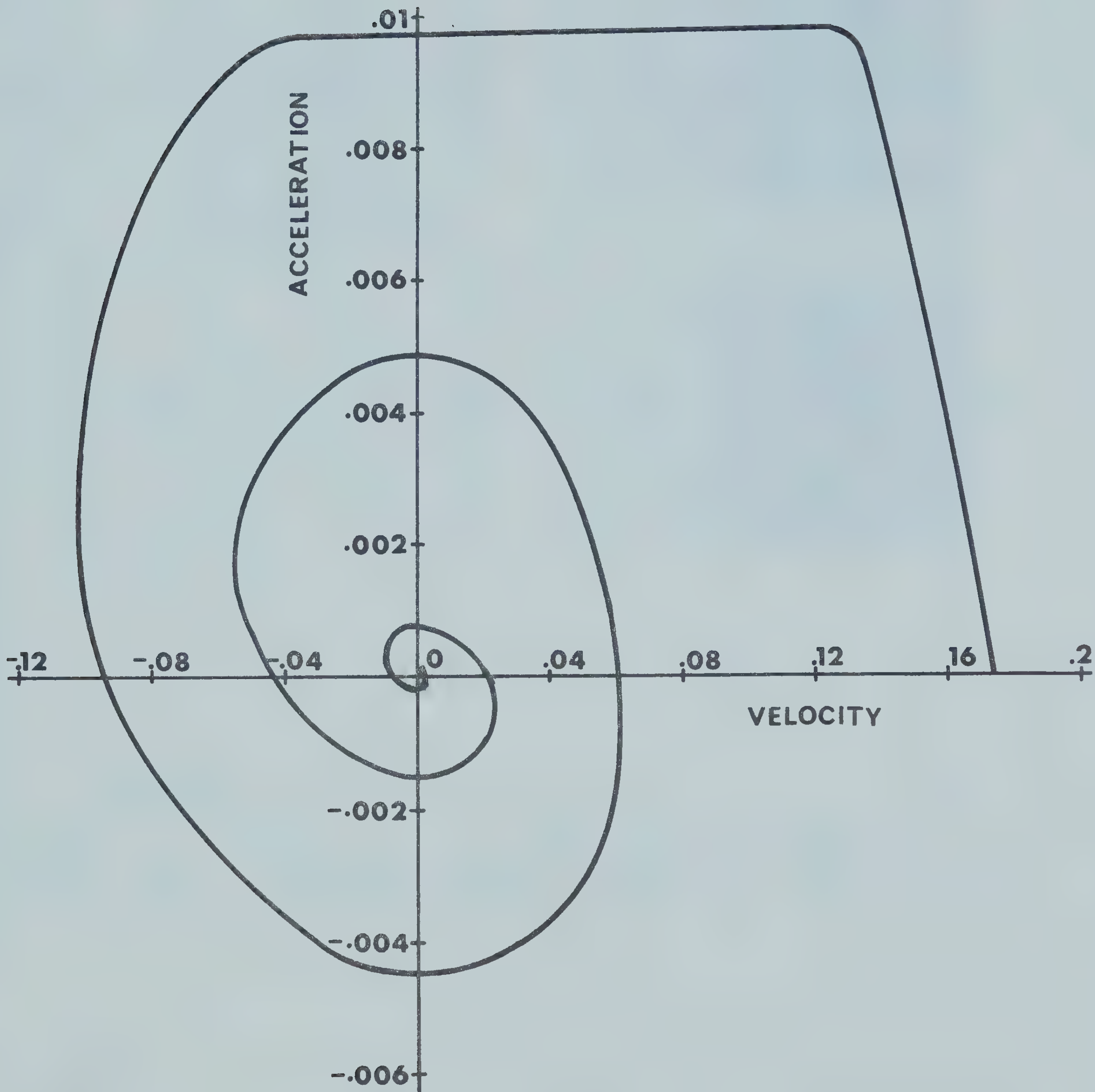

Fig. 4.8

Velocity vs acceleration corresponding to Fig. 4.6

100% torque pulse (case c, proportional control)

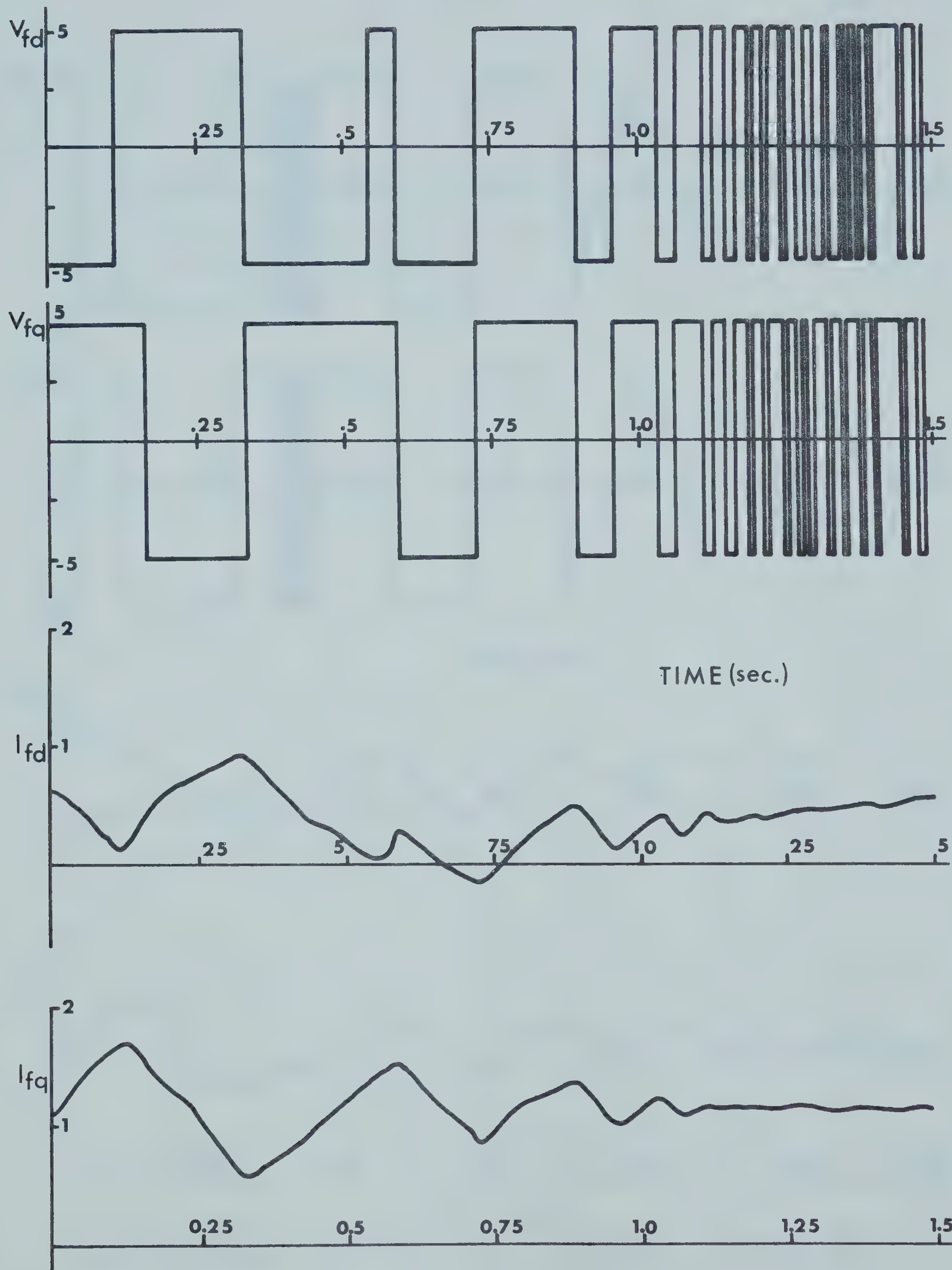

Fig. 4.9

Field voltages and field currents time characteristics corresponding to Fig. 4.6 100% torque pulse (case b, bang-bang control)

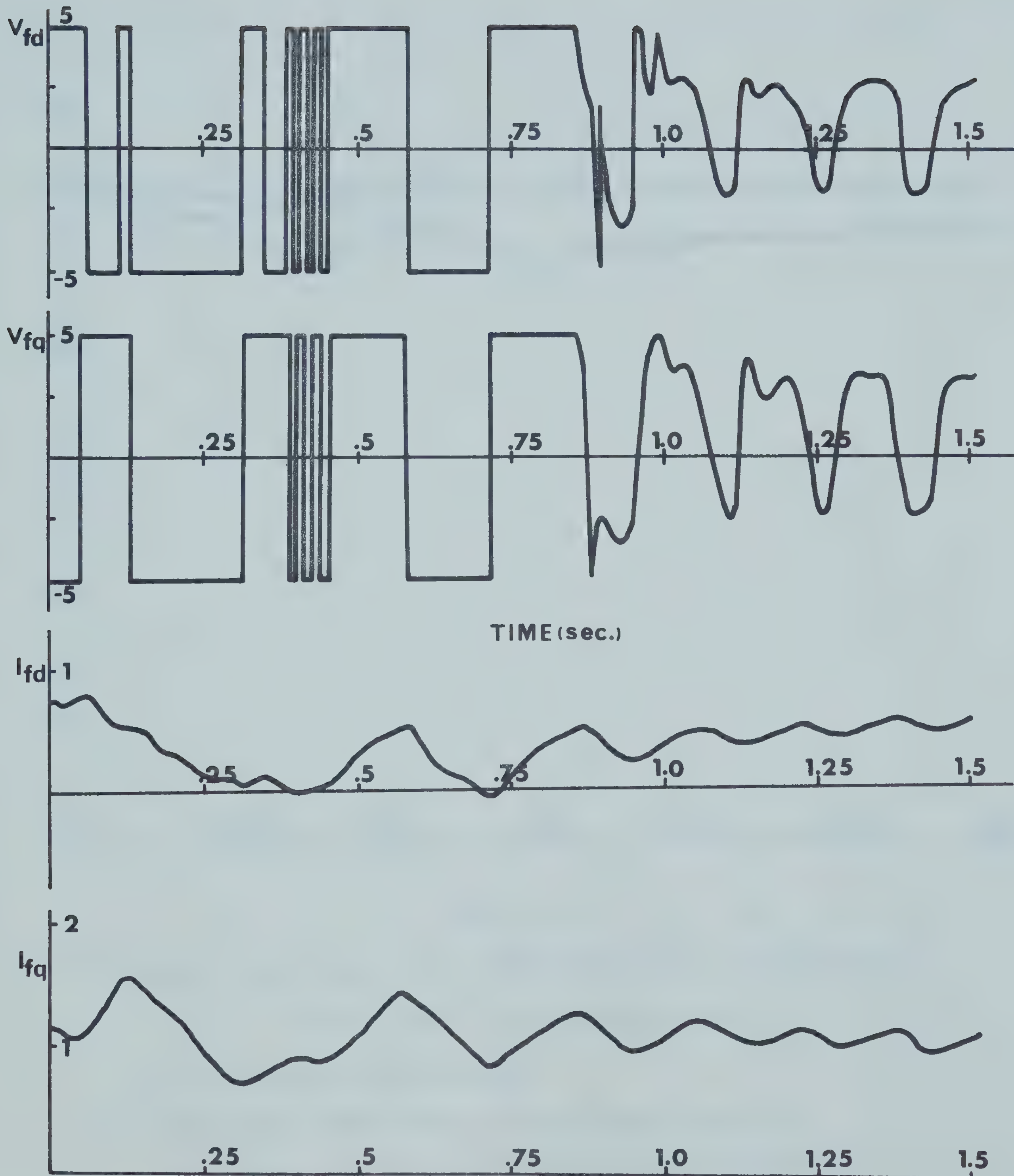

Fig. 4.10

Field voltages and field currents characteristics corresponding to Fig. 4.6 100% torque pulse (case c, proportional control)

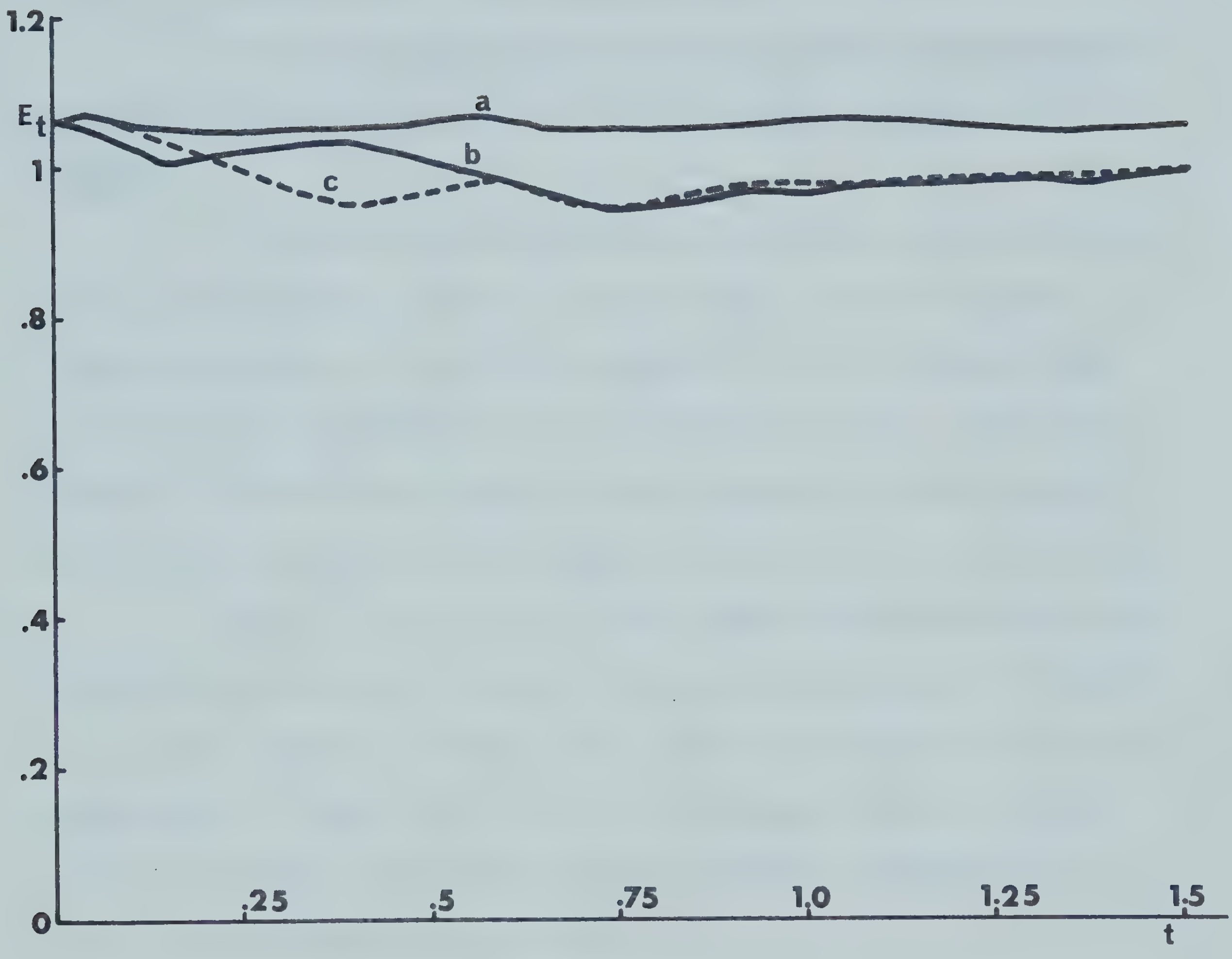

Fig. 4.11

Terminal voltages time characteristics corresponding to Fig. 4.6 100% torque pulse

a) unregulated machine

b) regulated machine (bang-bang control)

c) regulated machine (proportional control)

because the time optimal policy was defined to bring system to a stable angle in a minimum time.

3. The Combination of the Voltage Regulator and the Angle Regulator with the Proportional Stabilizing Signal

Practical considerations require a constant angle value and voltage within a given range. This requires implementation of a voltage regulator on the direct axis and an angle regulator on the quadrature axis. By introducing a proportional stabilizing signal on both axes of the rotor the final rotor angle is reached in shorter time.

Fig. 4.12-b is an angle time characteristics for a 100% torque puls. The new final stable angle is reached in a short period of time with only one overshoot and one undershoot. From Fig. 4.14-b it is seen that the variation of terminal voltage is much improved (compare with Fig. 4.11-b bang-bang control).

For the same system and for the same disturbance an angle regulator on the quadrature axis is added to bring the system back to its initial rotor position. Fig. 4.12-c shows that after the disturbance is cleared the initial rotor angle is regained.

From Fig. 4.14 it can be seen that the terminal voltage is initially deteriorated by the strong stabilizing signal.

At the initial time, variations of field voltage are almost bang-bang (Fig. 4.15) but after the stable point is reached the voltage variation on direct axis is smoother and on the quadrature axis is still big due to the proportional signal presented on quadrature axis.

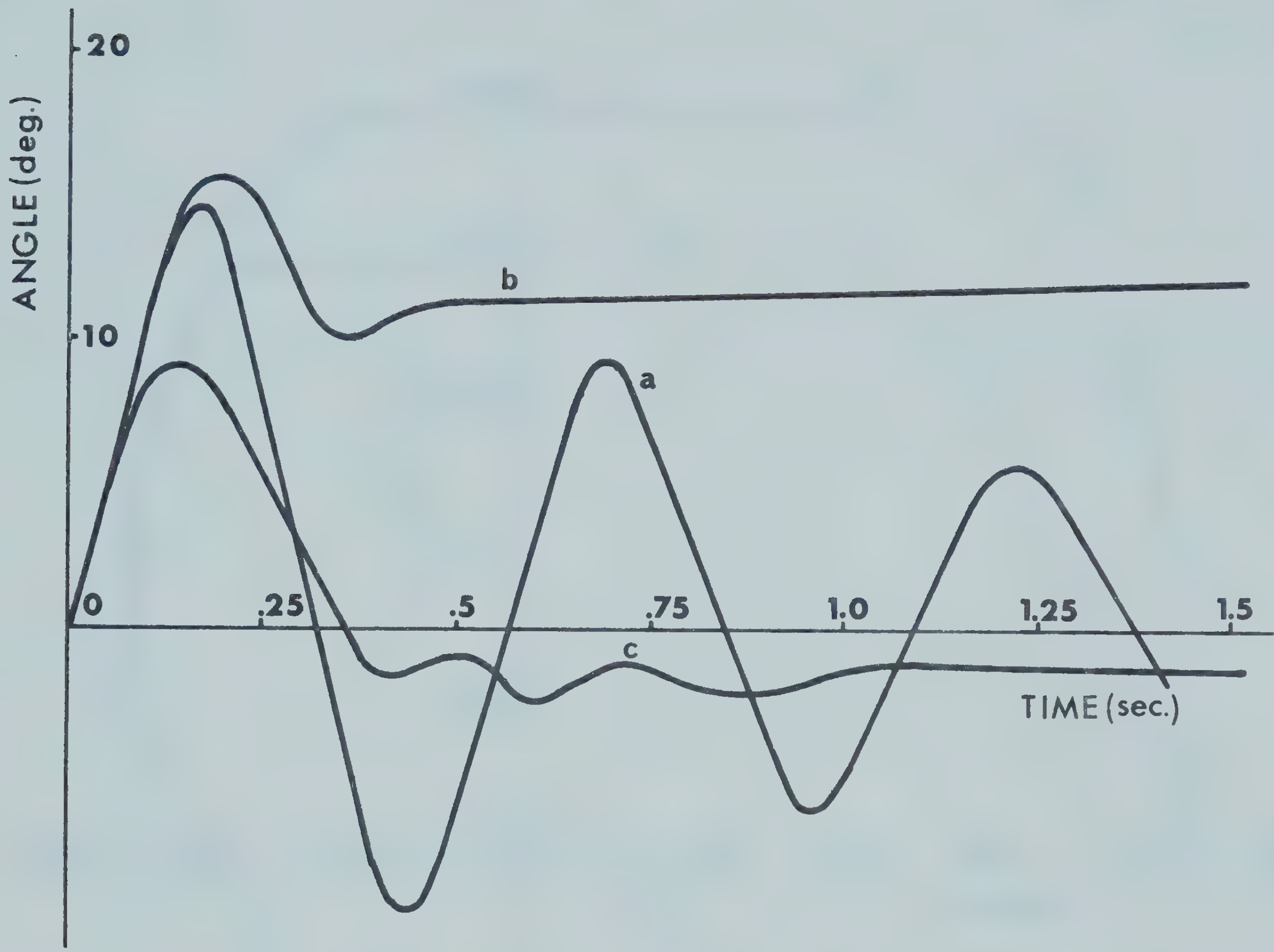

Fig. 4.12

Angle time characteristics for 100% torque pulse (3 cycles)

a) unregulated machine

b) regulated machine with a proportional control on quadrature axis and voltage regulator on direct axis.

c) regulated machine with an angle regulator on quadrature axis, voltage regulator on direct axis and stabilizing signal on both axis.

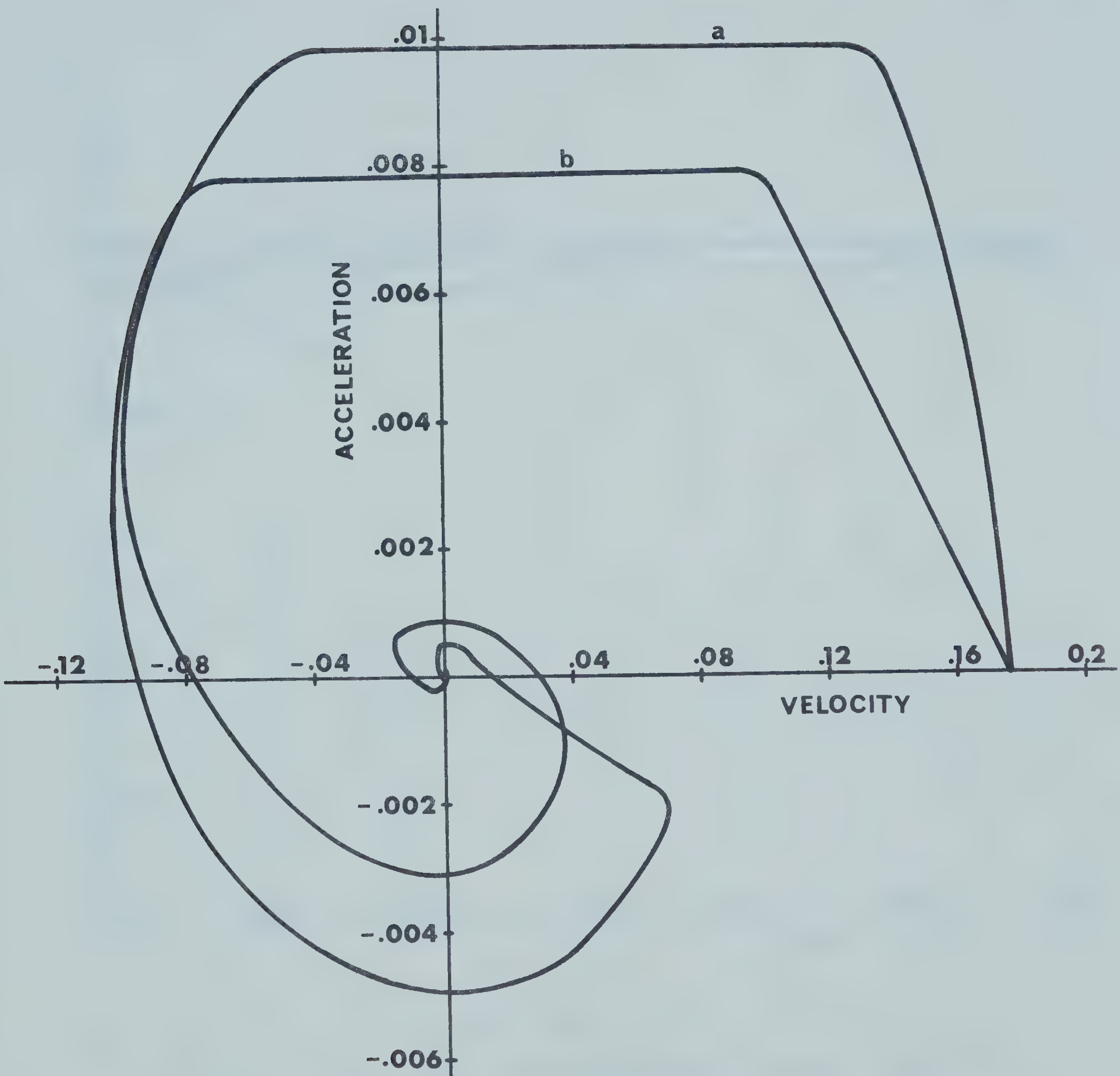

Fig. 4.13

Velocity vs acceleration plots corresponding to Fig. 4.12

a) regulated machine with voltage regulator on direct axis and proportional control on quadrature axis

b) regulated machine with voltage and angle regulators plus stabilizing signal

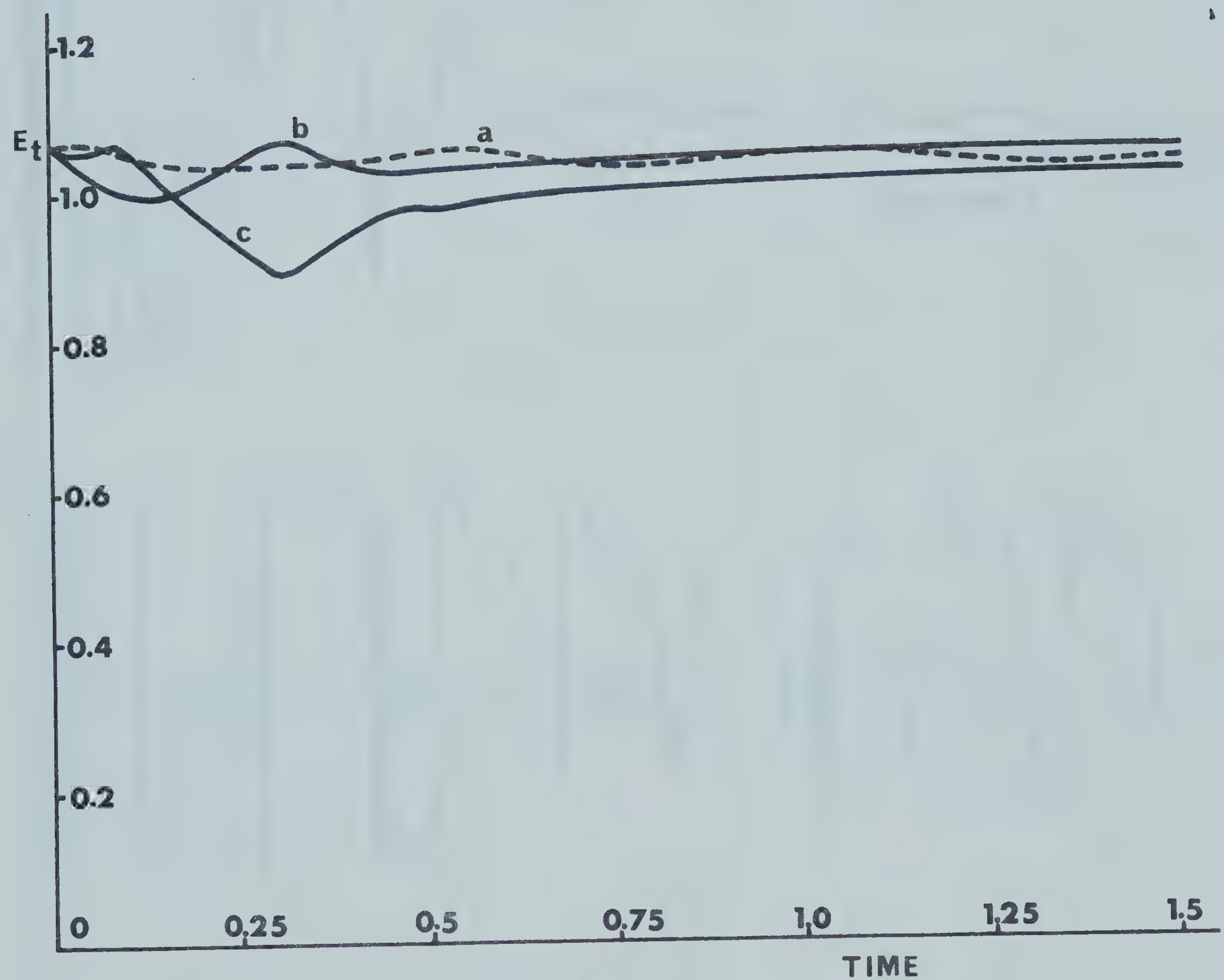

Fig. 4.14

Terminal voltages plot corresponding to Fig. 4.12

a) unregulated machine

b) regulated machine with voltage regulator on direct axis and proportional control on quadrature axis.

c) regulated machine with voltage and angle regulators plus stabilizing signal on both axis

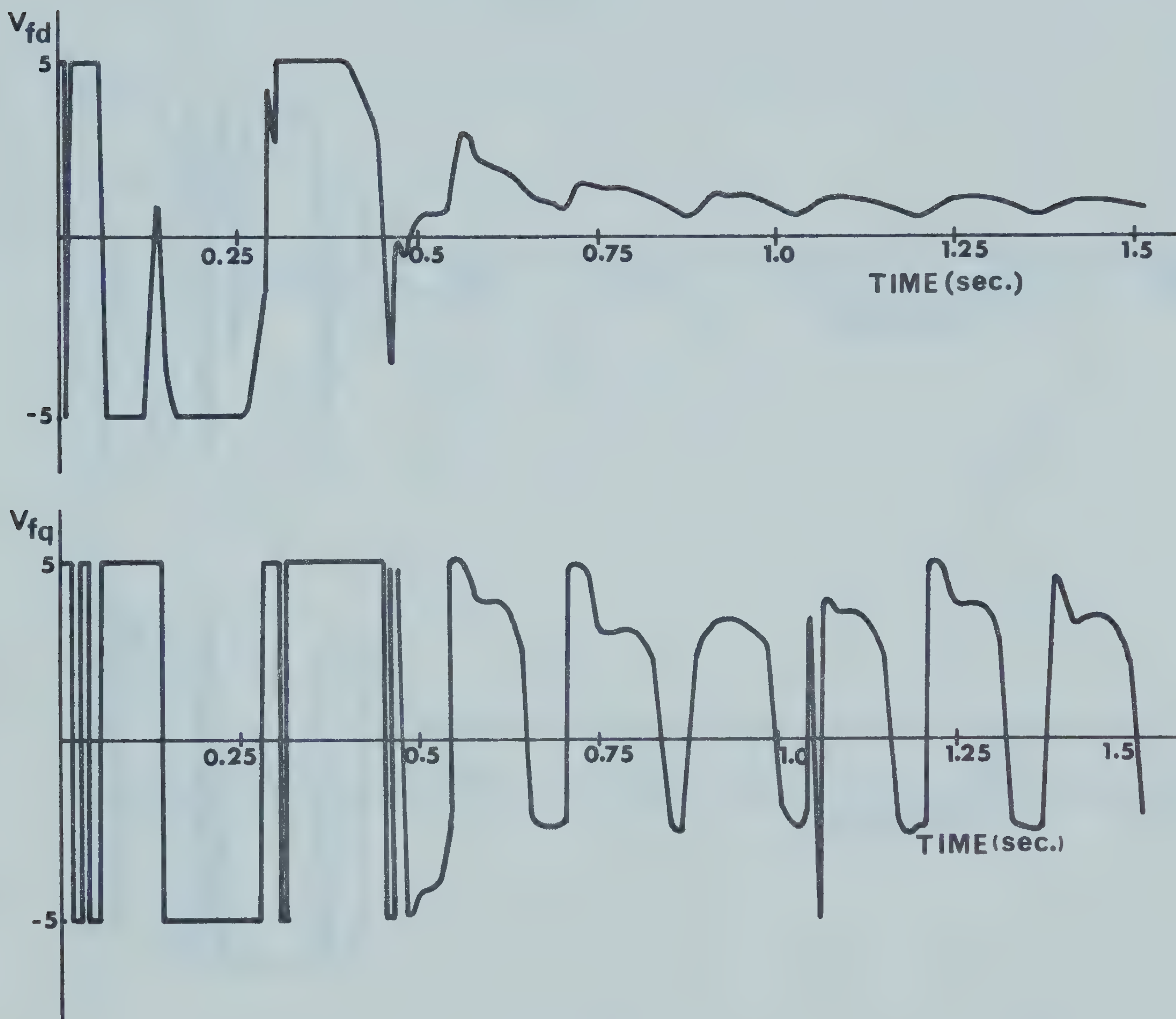

Fig. 4.15

Field voltages variation for 100% pulse (3 cycles) corresponding to Fig. 4.12 (case b, voltage regulator on direct axis and proportional signal on quadrature axis.

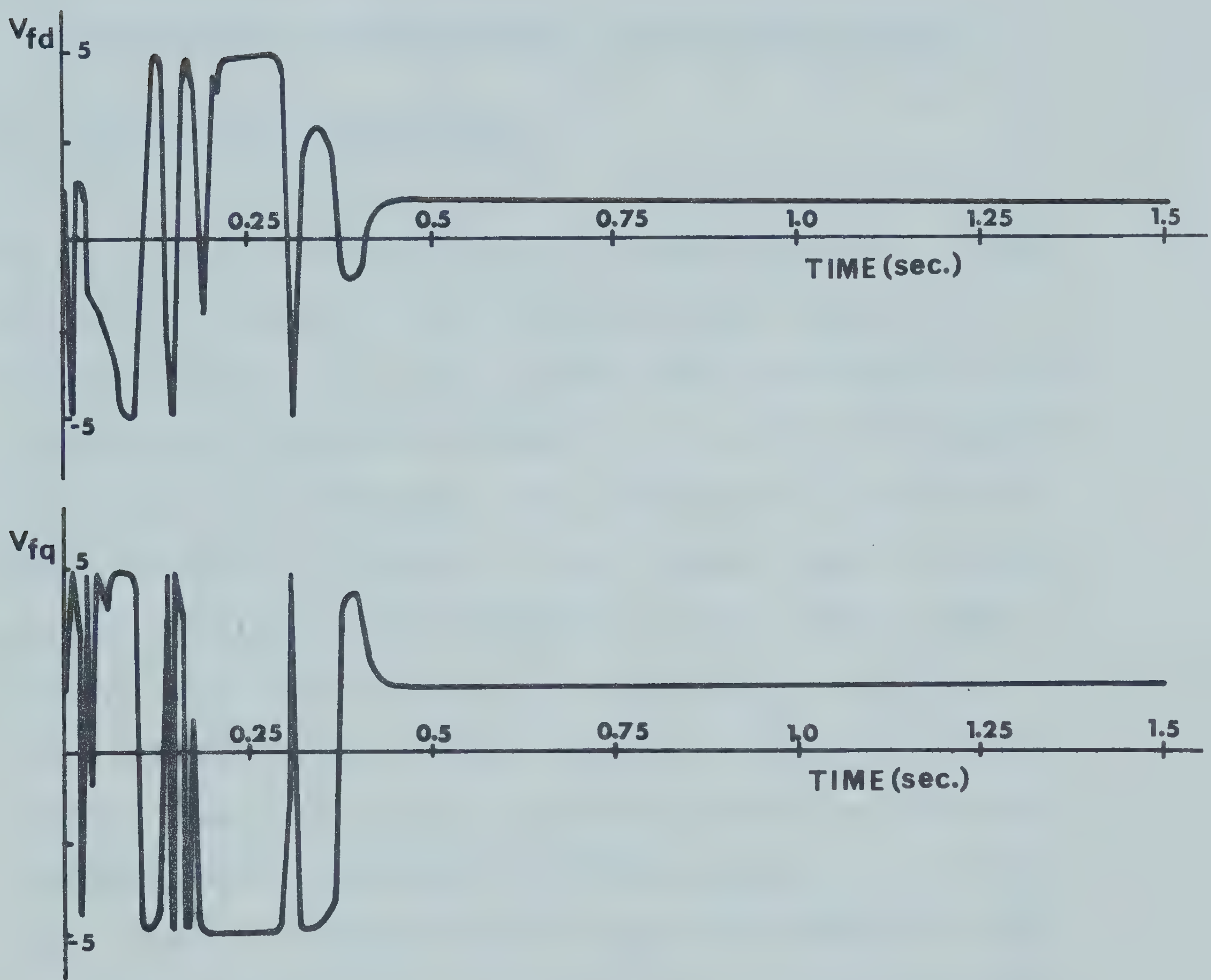

Fig. 4.16

Field voltages variation for 100% torque pulse (3 cycles) corresponding to Fig. 3.12 (case c, voltage regulator on direct axis and angle regulator on quadrature axis with proportional signal on both axis). After reaching stable angle the proportional signal is switched off.

CHAPTER 5

CONCLUSIONS AND SUGGESTIONS FOR FURTHER RESEARCH

5.1 Summary and Conclusions

This thesis presents a study of the transient stability of a dual-exciter synchronous generator. Optimal control theory in the closed form was applied to a nonlinear model. It can be noted that the unknown initial values of the costate variables π_1, π_2 and π_3 (in equation 3.22) were not calculated. The sub-optimal control was found solely as a function of the states. Such an implementation requires the measurement of the entire state vector. The result of this is a bang-bang control law, i.e., control actions switch between extremes of allowed values. Bang-bang control lacks the ability to cope with feedback control required for other purposes. As a practical solution a proportional control in combination with a voltage regulator on the direct axis and an angle regulator on the quadrature axis is proposed.

The significance of the analysis of this nonlinear model is of importance because it gives a correct representation of the system in any operating condition. Control of such a system is difficult especially with single control. The analysis of the results shows that the time optimal responses were not achieved. In general

sub-optimal solutions were obtained. The time required for stabilization was less than one second compared to a time of three to four seconds obtained using conventional regulators with first and second derivative feedback stabilizing signals.

The double winding rotor machine while somewhat more expensive than a conventional machine has significant operating advantage. Use of sub-optimal closed form excitation as described in this paper can provide stabilization at a cost which is small compared to the costs of presently used alternatives.

With judicious choice of approximate control signals the control complexity can be reduced to little more than that of conventional excitation control systems.

5.2 Suggestion for Further Research

One factor influencing the performance of the system is the uncontrollable initial periods. This depends on the severity of the disturbance. Further study on modifications of the control may reduce or eliminate this. This will increase the complexity of the system. Another area for further study is that of obtaining simple satisfactory practical control signals. That is, a control using readily measured variables.

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NOTATION

V_{fd}	Direct-axis field voltage
V_{fq}	Quadrature-axis field voltage
V_d	Direct-axis bus voltage
V_q	Quadrature-axis bus voltage
e_d	Direct-axis armature voltage
e_q	Quadrature-axis armature voltage
r_{fd}	Direct-axis field resistance
r_{fq}	quadrature-axis field resistance
r_{Kd}	Direct-axis amortisseur resistance
r_{Kq}	Quadrature-axis amortisseur resistance
R	Armature resistance
x_d	Direct-axis armature reactance
x_q	Quadrature-axis armature reactance
x_{afd}	Direct-axis mutual reactance between armature and field
x_{afq}	Quadrature-axis mutual reactance between armature and field
x_{akd}	Direct-axis mutual reactance between amortisseur and field
x_{akq}	Quadrature-axis mutual reactance between amortisseur and field
x_{ffd}	Direct-axis field reactance
x_{ffq}	Quadrature-axis field reactance

x_{kkd}	Direct-axis amortisseur reactance
x_{kkq}	Quadrature-axis amortisseur reactance
X_L	Line reactance
ψ_d	Direct-axis armature flux linkage
ψ_q	Quadrature-axis armature flux linkage
ψ_{fd}	Direct-axis field flux linkage
ψ_{fq}	Quadrature-axis field flux linkage
ψ_{kd}	Direct-axis amortisseur flux linkage
ψ_{kq}	Quadrature-axis amortisseur flux linkage
ω_o	Synchronous speed in radians per second
p	Differential operator
T_j	Inertia constant in seconds
δ	Rotor angle with reference to infinite bus in radians
T_t	Turbine torque
T_{el}	Electromagnetic torque
K_d	Mechanical damping coefficient
T_v	Voltage regulator time constant
T_a	Angle regulator time constant
K_v	Voltage regulator gain
K_a	Angle regulator gain
i_{fd}	Direct-axis field current
i_{fq}	Quadrature-axis field current
i_d	Direct-axis armature current
i_q	Quadrature-axis armature current
$u(t)$	Stabilizing signal
Σ	Switching function

APPENDIX I

The A, B, E and F coefficients are:

$A_{11} = \frac{1}{\omega_o} x_{afd}$ $A_{21} = \frac{1}{\omega_o} x_{ffd}$ $A_{31} = \frac{1}{\omega_o} x_{fkd}$

$A_{12} = \frac{1}{\omega_o} x_{akd}$ $A_{22} = \frac{1}{\omega_o} x_{fkd}$ $A_{32} = \frac{1}{\omega_o} x_{kkd}$

$A_{13} = -(\frac{1}{\omega_o} x_d + L_e)$ $A_{23} = \frac{1}{\omega_o} x_{afd}$ $A_{33} = -\frac{1}{\omega_o} x_{akd}$

$B_{13} = R_e + R$ $B_{21} = -r_{fd}$

$B_{14} = x_{afq}$ $B_{32} = -r_{kd}$

$B_{15} = x_{akq}$

$B_{16} = -(x_q + x_e)$

$E_{11} = \frac{1}{\omega_o} x_{afq}$ $E_{21} = \frac{1}{\omega_o} x_{ffq}$ $E_{31} = \frac{1}{\omega_o} x_{fkq}$

$E_{12} = \frac{1}{\omega_o} x_{akq}$ $E_{22} = \frac{1}{\omega_o} x_{fkq}$ $E_{32} = \frac{1}{\omega_o} x_{kkq}$

$E_{13} = -(\frac{1}{\omega_o} x_q + L_e)$ $E_{23} = -\frac{1}{\omega_o} x_{afq}$ $E_{33} = -\frac{1}{\omega_o} x_{akq}$

$F_{41} = -x_{afd}$

$F_{42} = -x_{akd}$

$F_{43} = (x_d + x_e)$

$F_{46} = R + R_e$

$F_{59} = -R_{fq}$

$F_{65} = -R_{kq}$

The A(I,J) coefficients are:

$A(1,1) = C_{12}\ B_{21}$

$A(1,2) = C_{13}\ B_{32}$

$A(1,3) = C_{11}\ B_{13}$

$A(1,4) = C_{11}\ B_{14}$

$A(1,5) = C_{11}\ B_{15}$

$A(1,6) = C_{11}\ B_{16}$

$A(2,1) = C_{22}\ B_{21}$

$A(2,2) = C_{23}\ B_{32}$

$A(2,3) = C_{21}\ B_{13}$

$A(2,4) = C_{21}\ B_{14}$

$A(2,5) = C_{21}\ B_{15}$

$A(2,6) = C_{21}\ B_{16}$

$A(4,1) = D_{11}\ F_{41}$

$A(4,2) = D_{11}\ F_{42}$

$A(4,3) = D_{11}\ F_{43}$

$A(4,4) = D_{12}\ F_{54}$

$A(4,5) = D_{13}\ F_{65}$

$A(4,6) = D_{11}\ F_{46}$

$A(5,1) = D_{21}\ F_{41}$

$A(5,2) = D_{21}\ F_{42}$

$A(5,3) = D_{21}\ F_{43}$

$A(5,4) = D_{22}\ F_{54}$

$A(5,5) = D_{23}\ F_{65}$

$A(5,6) = D_{21}\ F_{46}$

$A(3,1) = C_{32}\ B_{21}$ $\qquad$ $A(6,1) = D_{31}\ F_{41}$

$A(3,2) = C_{33}\ B_{32}$ $\qquad$ $A(6,2) = D_{31}\ F_{42}$

$A(3,3) = C_{31}\ B_{13}$ $\qquad$ $A(6,3) = D_{31}\ F_{43}$

$A(3,4) = C_{31}\ B_{14}$ $\qquad$ $A(6,4) = D_{32}\ F_{54}$

$A(3,5) = C_{31}\ B_{15}$ $\qquad$ $A(6,5) = D_{33}\ F_{65}$

$A(3,6) = C_{31}\ B_{16}$ $\qquad$ $A(6,6) = D_{31}\ F_{46}$

where the D and C coefficients are given on next page.

The C and D coefficients are:

$$C_{11} = (A_{22}\,A_{33} - A_{32}A_{23})/A_M$$
$$C_{12} = (A_{32}\,A_{13} - A_{12}\,A_{33})/A_M$$
$$C_{13} = (A_{12}\,A_{23} - A_{22}\,A_{13})/A_M$$
$$C_{21} = (A_{31}\,A_{23} - A_{21}\,A_{33})/A_M$$
$$C_{22} = (A_{11}\,A_{33} - A_{31}\,A_{13})/A_M$$
$$C_{23} = (A_{21}\,A_{13} - A_{11}\,A_{23})/A_M$$
$$C_{31} = (A_{21}\,A_{32} - A_{31}\,A_{22})/A_M$$
$$C_{32} = (A_{31}\,A_{12} - A_{11}\,A_{32})/A_M$$
$$C_{23} = (A_{11}\,A_{22} - A_{21}\,A_{12})/A_M$$

$$D_{11} = (E_{22}\,E_{33} - E_{32}\,E_{23})/E_M$$
$$D_{12} = (E_{32}\,E_{13} - E_{12}\,E_{33})/E_M$$
$$D_{13} = (E_{12}\,E_{23} - E_{22}\,E_{13})/E_M$$
$$D_{21} = (E_{31}\,E_{23} - E_{21}\,E_{33})/E_M$$
$$D_{22} = (E_{11}\,E_{33} - E_{31}\,E_{13})/E_M$$
$$D_{23} = (E_{21}\,E_{13} - E_{11}\,E_{23})/E_M$$
$$D_{31} = (E_{21}\,E_{32} - E_{31}\,E_{22})/E_M$$
$$D_{32} = (E_{31}\,E_{12} - E_{11}\,E_{32})/E_M$$
$$D_{33} = (E_{11}\,E_{22} - E_{21}\,E_{12})/E_M$$

where the $A_M = A_{11}\,A_{22}\,A_{33} + A_{12}\,A_{23}\,A_{31} + A_{13}\,A_{21}\,A_{32}$
$\qquad - A_{31}\,A_{22}\,A_{13} - A_{32}\,A_{23}\,A_{11} - A_{33}\,A_{21}\,A_{12}$

and $E_M = E_{11}\,E_{22}\,E_{33} + E_{12}\,E_{23}\,E_{31} + E_{13}\,E_{21}\,E_{32}$
$\qquad - E_{31}\,E_{22}\,E_{13} - E_{32}\,E_{23}\,E_{11} - E_{33}\,E_{21}\,E_{12}$

System Parameters

The parameters are of a 200 Mw, 16.5 KV turbo-generator which has a quadrature-axis field winding identical with the direct-axis field winding.

$x_d = 1.6$ p.u.	$x_{afd} = 1.45$ p.u.
$x_q = 1.6$ p.u.	$x_{afq} = 1.45$ p.u.
$x_{ffd} = 1.6$ p.u.	$R_{fd} = 0.001$ p.u.
$x_{ffq} = 1.6$ p.u.	$R_{fq} = 0.001$ p.u.
$x_{fkd} = 1.45$ p.u.	$R_e = 0.1$ p.u.
$x_{fkq} = 1.45$ p.u.	$x_e = 0.3$ p.u.
$x_{kkd} = 1.51$ p.u.	$K_d = 0.004$ p.u.
$x_{kkq} = 1.51$ p.u.	$H = 3.0$ p.u.
$x_{akq} = 1.45$ p.u.	$V = 1.0$ p.u.
$R_{kd} = 0.009$ p.u.	
$R_{kq} = 0.009$ p.u.	

The operating point calculated from 2.35 is given by:

$P_o = 1.0$ p.u.	$V_{fd} = 1.052$ p.u.	$E_t = 1.061$ p.u.
$Q_o = 0.0$ p.u.	$V_{fq} = 1.75$ p.u.	
$\delta = 0.0$ rad.	$V = 1.0$	

APPENDIX II

Equation 3.5 is

$$
\begin{aligned}
pT_e = {} & x_{afd}\, i_q\, pi_{fd} + x_{akd}\, i_q\, pi_{kd} \\
& + x_{ffd}\, i_{fd}\, pi_q + x_{fkd}\, i_{kd}\, pi_q \\
& - x_{afq}\, i_d\, pi_{fq} - x_{akq}\, i_d\, pi_{kq} \\
& - x_{afq}\, i_{fq}\, pi_d - x_{akq}\, i_{kq}\, pi_d
\end{aligned}
$$

Substituting expression for derivatives in the above equation, yields:

$$
\begin{aligned}
pT_e = {} & x_{afd}\, i_q\, A(1,1)\, i_{fd} && x_{akd}\, i_q\, A(2,1)\, i_{fd} \\
& + x_{afd}\, i_q\, A(1,2)\, i_{kd} && + x_{akd}\, i_q\, A(2,2)\, i_{kd} \\
& + x_{afd}\, i_q\, A(1,3)\, i_d && + x_{ald}\, i_q\, A(2,3)\, i_d \\
& + x_{afd}\, i_q\, A(1,4)\, i_{fq} && + x_{akd}\, i_q\, A(2,4)\, i_{fq} \\
& + x_{afd}\, i_q\, A(1,4)\, i_{fq}\, n && + x_{akd}\, i_q\, A(2,4)\, i_{fq}\, n \\
& + x_{afd}\, i_q\, A(1,5)\, i_{kq} && + x_{akd}\, i_q\, A(2,5)\, i_{kq} \\
& + x_{afd}\, i_q\, A(1,5)\, i_{kq}\, n && + x_{akd}\, i_q\, A(2,5)\, i_{kq}\, n \\
& + x_{afd}\, i_q\, A(1,6)\, i_q && + x_{akd}\, i_q\, A(2,6)\, i_q \\
& + x_{afd}\, i_q\, A(1,6)\, i_q\, n && + x_{akd}\, i_q\, A(2,6)\, i_q\, n \\
& + x_{afd}\, i_q\, C_{11}\, V \sin\delta && + x_{akd}\, i_q\, C_{21}\, V \sin\delta \\
& + x_{afd}\, i_q\, C_{12}\, V_{fd} + && + x_{akd}\, i_q\, C_{22}\, V_{fd}
\end{aligned}
$$

$$+ (x_{ffd}\ i_{fd} + x_{fkd}\ i_{kd})\ A(6,1)\ i_{fd}$$
$$+ (x_{ffd}\ i_{fd} + x_{fkd}\ i_{kd})\ A(6,1)\ i_{fd}\ n$$
$$+ (x_{ffd}\ i_{fd} + x_{fkd}\ i_{kd})\ A(6,2)\ i_{kd}$$
$$+ (x_{ffd}\ i_{fd} + x_{fkd}\ i_{kd})\ A(6,2)\ i_{kd}\ n$$
$$+ (x_{ffd}\ i_{fd} + x_{fkd}\ i_{kd})\ A(6,3)\ i_{d}$$
$$+ (x_{ffd}\ i_{fd} + x_{fkd}\ i_{kd})\ A(6,3)\ i_{d}\ n$$
$$+ (x_{ffd}\ i_{fd} + x_{f\ d}\ i_{kd})\ A(6,4)\ i_{fq}$$
$$+ (x_{ffd}\ i_{fd} + x_{fkd}\ i_{kd})\ A(6,5)\ i_{kq}$$
$$+ (x_{ffd}\ i_{fd} + x_{fkd}\ i_{kd})\ A(6,6)\ i_{q}$$
$$+ (x_{ffd}\ i_{fd} + x_{fkd}\ i_{kd})\ D_{31}\ V\cos\delta$$
$$+ (x_{ffd}\ i_{fd} + x_{fkd}\ i_{kd})\ D_{32}\ V_{fq}$$

$$- x_{afq}\ i_d\ A(4,1)\ i_{fd}$$
$$- x_{afq}\ i_d\ A(4,1)\ i_{fd}\ n$$
$$- x_{afq}\ i_d\ A(4,2)\ i_{kd}$$
$$- x_{afq}\ i_d\ A(4,2)\ i_{kd}\ n$$
$$- x_{afq}\ i_d\ A(4,3)\ i_{d}$$
$$- x_{afq}\ i_d\ A(4,3)\ i_{d}\ n$$
$$- x_{afq}\ i_d\ A(4,4)\ i_{fq}$$
$$- x_{afq}\ i_d\ A(4,5)\ i_{kq}$$
$$- x_{afq}\ i_d\ A(4,6)\ i_{q}$$
$$- x_{afq}\ i_d\ D_{11}\ V\cos\delta$$
$$- x_{afq}\ i_d\ D_{12}\ V_{fq}$$

$$- x_{akq}\ i_d\ A(5,1)\ i_{fd}$$
$$- x_{akq}\ i_d\ A(5,1)\ i_{fd}\ n$$
$$- x_{akq}\ i_d\ A(5,2)\ i_{kd}$$
$$- x_{akq}\ i_d\ A(5,2)\ i_{kd}\ n$$
$$- x_{akq}\ i_d\ A(5,3)\ i_{d}$$
$$- x_{akq}\ i_d\ A(5,3)\ i_{d}\ n$$
$$- x_{akq}\ i_d\ A(5,4)\ i_{fq}$$
$$- x_{akq}\ i_d\ A(5,5)\ i_{kq}$$
$$- x_{akq}\ i_d\ A(5,6)\ i_{q}$$
$$- x_{akq}\ i_d\ D_{21}\ V\cos\delta$$
$$- x_{akq}\ i_d\ D_{22}\ V_{fq}$$

$$- (x_{afq}\ i_{fq} + x_{akq}\ i_{kq})\ A(3,1)\ i_{fd}$$
$$- (x_{afq}\ i_{fq} + x_{akq}\ i_{kq})\ A(3,2)\ i_{kd}$$
$$- (x_{afq}\ i_{fq} + x_{akq}\ i_{kq})\ A(3,3)\ i_{d}$$
$$- (x_{afq}\ i_{fq} + x_{akq}\ i_{kq})\ A(3,4)\ i_{fq}$$
$$- (x_{afq}\ i_{fq} + x_{akq}\ i_{kq})\ A(3,4)\ i_{fq}\ n$$
$$- (x_{afq}\ i_{fq} + x_{akq}\ i_{kq})\ A(3,5)\ i_{kq}$$
$$- (x_{afq}\ i_{fq} + x_{akq}\ i_{kq})\ A(3,5)\ i_{kq}\ n$$
$$- (x_{afq}\ i_{fq} + x_{akq}\ i_{kq})\ A(3,6)\ i_{q}$$
$$- (x_{afq}\ i_{fq} + x_{akq}\ i_{kq})\ A(3,6)\ i_{q}\ n$$
$$- (x_{afq}\ i_{fq} + x_{akq}\ i_{kq})\ C_{31}\ V\sin\delta$$
$$- (x_{afq}\ i_{fq} + x_{akq}\ i_{kq})\ C_{32}\ V_{fd} \qquad \text{(A2-1)}$$

Equation A2-1 can be written as

$$pT_e = -\ (E + V_{fd}\ BN + V_{fq}\ BK)$$

where

$$BN(t) = x_{afd}\ i_q\ C_{12} + x_{akd}\ i_q\ C_{22} - x_{afq}\ C_{32} - x_{adq}\ i_{kq}\ C_{32}$$

$$BK(t) = x_{ffd}\ i_{fd}\ D_{32} + x_{fkd}\ i_{kd}\ D_{32} - x_{afq}\ i_d\ D_{12} - x_{akq}\ i_d\ D_{22}$$

$E(t) =$

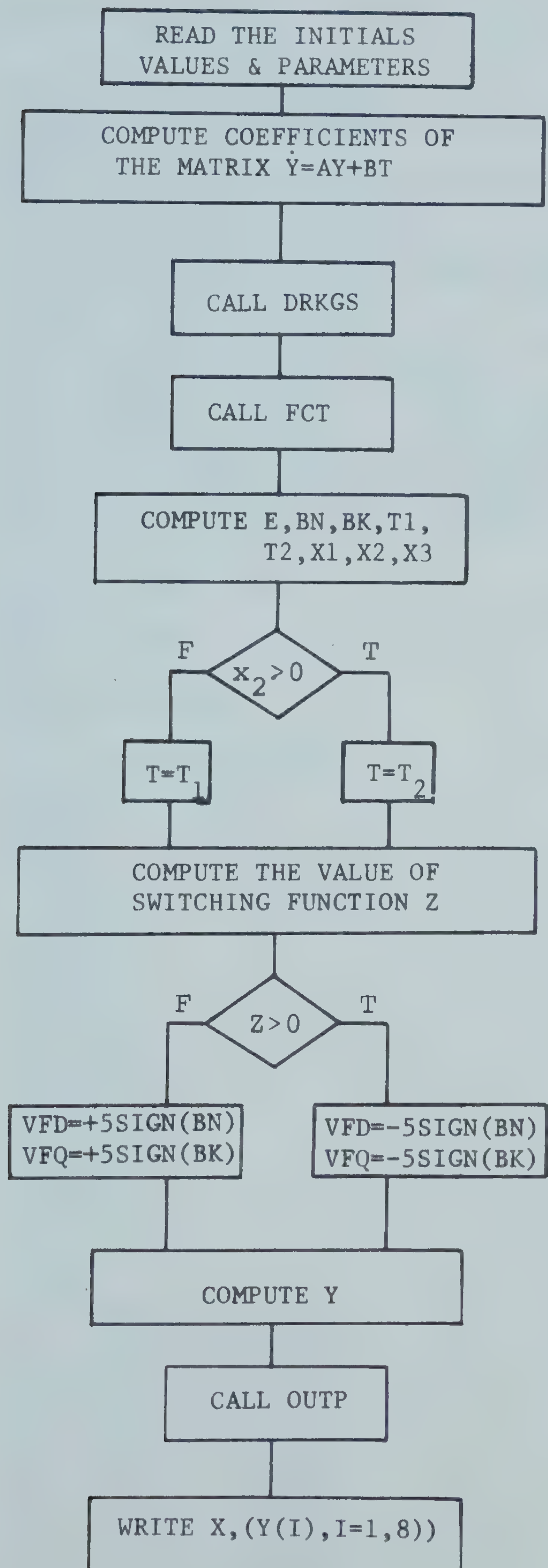
$$x_{afd}\ i_q\ A(1,1)\ i_{fd} + x_{afd}\ i_q\ A(1,2)\ i_{kd} + x_{afd}\ i_q\ A(1,3)\ i_d$$
$$+x_{afd}\ i_q\ A(1,4)\ i_{fq} + x_{afd}\ i_q\ A(1,5)\ i_{kq} + x_{afd}\ i_q\ A(1,6)\ i_q$$

$$+ x_{afd}\, i_q\, C_{11}\, V_d + x_{akd}\, i_q\, A(2,1)\, i_{fd} + x_{akd}\, i_q\, A(2,2)\, i_{kd}$$
$$+ x_{akd}\, i_q\, A(2,3)\, i_d + x_{akd}\, i_q\, A(2,4)\, i_{fq} + x_{akd}\, i_q\, A(2,5)\, i_{kq}$$
$$+ x_{akd}\, i_q\, A(2,6)\, i_q + x_{akd}\, i_q\, C_{21}\, V \sin\delta - x_{afq}\, i_d\, A(4,1)\, i_{fd}$$
$$- x_{afq}\, i_d\, A(4,2)\, i_{kd} - x_{afq}\, i_d\, A(4,3)\, i_d - x_{afq}\, i_d\, A(4,4)\, i_{fq}$$
$$- x_{afq}\, i_d\, A(4,5)\, i_{kq} - x_{afq}\, i_d\, A(4,6)\, i_q - x_{afq}\, i_d\, D_{11}\, V \cos\delta$$
$$- x_{akq}\, i_d\, A(5,1)\, i_{fd} - x_{akq}\, i_d\, A(5,2)\, i_{kd} - x_{akq}\, i_d\, A(5,3)\, i_d$$
$$- x_{akq}\, i_d\, A(5,6)\, i_{fq} - x_{akq}\, i_d\, A(5,5)\, i_{kq} - x_{akq}\, i_d\, A(5,6)\, i_q$$
$$- x_{akq}\, i_d\, D_{21}\, V \cos\delta + (x_{ffd}\, i_{fd} + x_{fkd}\, i_{kd})\, A(6,1)\, i_{fd}$$
$$+ (x_{ffd}\, i_{fd} + x_{fkd}\, i_{kd})\, A(6,2)\, i_{kd}$$
$$+ (x_{ffd}\, i_{fd} + x_{fkd}\, i_{kd})\, A(6,3)\, i_d$$
$$+ (x_{ffd}\, i_{fd} + x_{fkd}\, i_{kd})\, A(6,4)\, i_{fq}$$
$$+ (x_{ffd}\, i_{fd} + x_{fkd}\, i_{kd})\, A(6,5)\, i_{kq}$$
$$+ (x_{ffd}\, i_{fd} + x_{fkd}\, i_{kd})\, A(6,6)\, i_q$$
$$+ (x_{ffd}\, i_{fd} + x_{fkd}\, i_{kd})\, D_{31}\, V \cos\delta$$
$$- (x_{afq}\, i_{fq} + x_{akq}\, i_{kq})\, A(3,1)\, i_{fd}$$
$$- (x_{afq}\, i_{fq} + x_{akq}\, i_{kq})\, A(3,2)\, i_{kd}$$
$$- (x_{afq}\, i_{fq} + x_{akq}\, i_{kq})\, A(3,3)\, i_d$$
$$- (x_{afq}\, i_{fq} + x_{akq}\, i_{kq})\, A(3,4)\, i_{fq}$$
$$- (x_{afq}\, i_{fq} + x_{akq}\, i_{kq})\, A(3,5)\, i_{kq}$$
$$- (x_{afq}\, i_{fq} + x_{akq}\, i_{kq})\, A(3,6)\, i_q$$
$$- (x_{afq}\, i_{fq} + x_{akq}\, i_{kq})\, C_{31}\, V \sin\delta$$

In the expression for E(t) the terms with n are neglected (order of n is 10^{-3}).

APPENDIX III

Computer Program



FORTRAN IV COMPILER (20.1)   MAIN   08-08-72

```
      IMPLICIT  REAL*8 (A-H,O-Z)
      DIMENSION  DERY(8),Y(8),PRMT(5),AUX(8,8)
      EXTERNAL FCT,OUTP
      COMMON RFD,XD,XE,XAFD,RE,R,XFFD,XFFQ,XQ,RFQ,XFKD,XFKQ,
     1XKKD,XKKQ,V,XAKD,XAKQ,RKD,RKQ
      COMMON C11,C12,C13,C21,C22,C23,C31,C32,C33,D11,D12,D13,D21,D22,
     1D23,D31,D32,D33,X1,X2,X3,X4,DD,G,Z1,Z2,T,Z
      COMMON A(6,6),C(3,4)
      COMMON ZL,XEOLD, REOLD,ZLOLD,VOLD,I,II,J
      COMMON /AA/VFD,VFQ,ET,BN,BK,E,S,Z3
      READ(5,33)Y(1),Y(2),Y(3),Y(4),Y(5),Y(6),Y(7),Y(8)
   33 FORMAT(1P2D20.10/1P2D20.10/1P2D20.10/1P2D20.10)
      READ(5,30)RFD,XD,XE,XAFD,XAFQ,RE,R,XFFD,XFFQ,XQ,RFQ,XFKD,XFKQ,
     1XKKD,XKKQ,V,DD,VFD,VFQ,XAKD,XAKQ,RKD,RKQ
   30 FORMAT (7D10.3/7D10.3/3D10.3/1P2D19,10/4D10.3)
      DO 2 K=1,8
    2 DERY(K)=1.0d0/8.0D0
      PRMT(1)=0.
      PRMT(2)=1.5D0
      PRMT(3)=0.001D0
      PRMT(4)=0.001D0
      NDIM=8
      G=314.0D0/6.0D0*0.004D0
      ZL=0.15D0/314.0D0
      X1=XAFD
      X2=XAKD
      X3=-XAFQ
      X4=-XAKQ
      B13=R+RE
      B14=XAFQ
      B15=XAKQ
      B16=-(XQ+XE)
      B21=-RFD
      B32=-RKD
      F41=-XAFD
      F42=-XAKD
      F43=XD+XE
      F46=R+RE
      F54=-RFQ
      F65=-RKQ
      A11=1.0D0/314.0D0*XAFD
      A12=1.0d0/314.0D0*XAKD
      A13=-1.0D0/314.0D0*XD
      A21=1.0D0/314.0D0*XFFD
      A22=1.0D0/314.0D0*XFKD
      A23=-1.0D0/314.0D0*XAFD
      A31=1.0D0/314.0D0*XFKD
      A32=1.0D0/314.0D0*XKKD
      A33=-1.0D0/314.0D0*XAKD
```

```
      AM=A11*A22*A33+A12*A23*A31+A13*A21*A32-A31*A22*A13-
     1A32*A23*A11-A33*A21*A12
      C11=(A22*A33-A32*A23)/AM
      C12=(A32*A13-A12*A33)/AM
      C13=(A12*A23-A22*A13)/AM
      C21=(A31*A23-A21*A33)/AM
      C22=(A11*A33-A31*A13)/AM
      C23=(A21*A13-A11*A23)/AM
      C31=(A21*A32-A31*A22)/AM
      C32=(A31*A12-A11*A32)/AM
      C33=(A11*A22-A21*A12)/AM
      E11=1.0D0/314.0D0*XAFQ
      E12=1.0D0/314.0D0*XAKQ
      E13=-1.0D0/314.0D0*XQ
      E21=1.0D0/314.0D0*XFFQ
      E22=1.0D0/314.0D0*XFKQ
      E23=-1.0D0/314.0D0*XAFQ
      E31=1.0D0/314.0D0*XFKQ
      E32=1.0D0/314.0D0*XKKQ
      E33=-1.0D0/414.0D0*XAKQ
      EM=E11*E22*E33+E12*E23*E31+E13*E21*E32-
     1E31*E22*E13-E32*E23*E11-E33*E21*E12
      D11=(E22*E33-E32*E23)/EM
      D12=(E32*E13-E12*E33)/EM
      D13=(E12*E23-E22*E13)/EM
      D21=(E31*E23-E21*E33)/EM
      D22=(E11*E33-E31*E13)/EM
      D23=(E21*E13-E11*E23)/EM
      D31=(E21*E32-E31*E22)/EM
      D32=(E31*E12-E11*E32)/EM
      D33=(E11*E22-E21*E12)/EM
      A(1,1)=C11*B21
      A(1,2)=C13*B32
      A(1,3)=C11*B13
      A(1,4)=C11*B14
      A(1,5)=C11*B15
      A(1,6)=C11*B16
      A(2,1)=C22*B21
      A(2,2)=C23*B32
      A(2,3)=C21*B13
      A(2,4)=C21*B14
      A(2,5)=C21*B15
      A(2,6)=C21*B16
      A(3,1)=C32*B21
      A(3,2)=C33*B32
      A(3,3)=C31*B13
      A(3,4)=C31*B14
      A(3,5)=C31*B15
      A(3,6)=C31*B16
      A(4,1)=D11*F41
      A(4,2)=D11*F42
```

```
0096      A(4,3)=D11*F43
0097      A(4,4)=D12*F54
0098      A(4,5)=D13*F65
0099      A(4,6)=D11*F46
0100      A(5,1)=D21*F41
0101      A(5,2)=D21*F42
0102      A(5,3)=D21*F43
0103      A(5,4)=D22*F54
0104      A(5,5)=D23*F65
0105      A(5,6)=D21*F46
0106      A(6,1)=D31*F41
0107      A(6,2)=D31*F42
0108      A(6,3)=D31*F43
0109      A(6,4)=D32*F54
0110      A(6,5)=D33*F65
0111      A(6,6)=D31*F46
0112      I=0
0113      CALL DRKGS (PRMT,Y,DERY,NDIM,IHLF,FCT,OUTP,AUX)
0114      IF(IHLF.GT.10) WRITE (6,66) IHLF
0115   66 FORMAT (1X,I3)
0116      STOP
0117      END


0001      SUBROUTINE FCT(X,Y,DERY)
0002      IMPLICIT REAL*8 (A-H,O-Z)
0003      DIMENSION DERY(8),Y(8),PRMT(5),AUX(8,8)
0004      COMMON RFD,XD,XE,XAFD,XAFQ,RE,R,XFFD,XFFQ,XQ,RFQ,XFKD,XFKQ,
         1XKKD,XKKQ,V,XAKD,XAKQ,RKD,RKQ
0005      COMMON C11,C12,C13,C21,C22,C23,C31,C32,C33,D11,D12,D13,D21,D22,
         1D23,D31,D33,X1,X2,X3,X4,DD,G,Z1,Z2,T,Z
0006      A(6,6),C(3,4)
0007      COMMON ZL,XEOLD,REOLD,ZLOLD,VOLD,I,II,J
0008      COMMON /AA/VFD,VFQ,ET,BN,E,S,Z3
0009      ED=RE*Y(3)-XE*Y(6)+ZL*DERY(3)+DSIN(Y8))*V
0010      EQ=RE*Y(6)+XE*Y(3)+ZL*DERY(6)+DCOS(Y8))*V
0011      ET=(ED**2+EQ**2)**0.5
0012      IF(X.LE.0.001D0) GO TO 100
0013      E=X1*Y(6)*A(1,1)*Y(1)+X1*Y(6)*A(1,2)*Y(2)+X1*Y(6)*A(1,3)*Y(3)+
         1X1*Y(6)*A(1,4)*Y(4)+X1*Y(6)*A(1,50*Y(5)+X1*Y(6)*A(1,6)*Y(6)+
         1X1*Y(1)*A(6,1)*Y(1)+X1*Y(1)*A(6,2)*Y(2)+X1*Y(1)*A(6,3)*Y(3)+
         1X1*Y(1)*A(6,4)*Y(4)+X1*Y(1)*A(6,5)*Y(5)+X1*Y(1)*A(6,6)*Y(6)+
         1X2*Y(6)*A(2,1)*Y(1)+X2*Y(6)*A(2,2)*Y(2)+X2*Y(6)*A(2,3)*Y(3)+
         1X2*Y(6)*A(2,4)*Y(4)+X2*Y(6)*A(2,5)*Y(5)+X2*Y(6)*A(2,6)*Y(6)+
         1X2*Y(2)*A(6,1)*Y(1)+X2*Y(2)*A(6,2)*Y(2)+X2*Y(2)*A(6,3)*Y(3)+
         1X2*Y(2)*A(6,4)*Y(4)+X2*Y(2)*A(6,5)*Y(5)+X2*Y(2)*A(6,6)*Y(6)+
         1X3*Y(3)*A(4,1)*Y(1)+X3*Y(3)*A(4,2)*Y(2)+X3*Y(3)*A(4,3)*Y(3)+
         1X3*Y(3)*A(4,4)*Y(4)+X3*Y(3)*A(4,5)*Y(5)+X3*Y(3)*A(4,6)*Y(6)+
         1X3*Y(4)*A(3,1)*Y(1)+X3*Y(4)*A(3,2)*Y(2)+X3*Y(4)*A(3,3)*Y(3)+
         1X3*Y(4)*A(3,4)*Y(4)+X3*Y(4)*A(3,5)*Y(5)+X3*Y(4)*A(3,6)*Y(6)+
         1X4*Y(3)*A(5,1)*Y(1)+X4*Y(3)*A(5,2)*Y(2)+X4*Y(3)*A(5,3)*Y(3)+
         1X4*Y(3)*A(5,4)*Y(3)+X4*Y(3)*A(5,5)*Y(5)+X4*Y(3)*A(5,6)*Y(6)+
```

```
          1X4*Y(5)*A(3,1)*Y(1)+X4*Y(5)*A(3,2)*Y(2)+X4*Y(5)*A(3,3)*Y(3)+
          1X4*Y(5)*A(3,4)*Y(4)+X4*Y(5)*A(3,5)*Y(5)+X4*Y(5)*A(3,6)*Y(6)+
          1(X1*Y(6)*C11+X2*Y(6)*C21+X3*Y(4)*C31+X4*Y(5)*C31)*V*DSIN(Y(8))+
          1(X1*Y(1)*D31+X2*Y(2)*D31+X3*Y(3)*D11+X4*Y(3)*D21)*V*DCOS(Y(8))
0014      BK=(X1*Y(1)*D32+X2*Y(2)*D32+X3*Y(3)*D12+X4*Y(3)*D22)*RFQ/XAFQ
0015      B1=DSIGN(1.0D0,BK)
0016      BN=(xL*Y(6)*C12+X2*Y(6)*C22+X3*Y(4)*C32+X4*Y(5)*C32)*RFD/XAFD
0017      B2=DSIGN(1.0D0,BN)
0018      T1=-314.0D0/6.0D0*(E-5.0D0*B2*BN-5.0D0*B1*BK)
0019      T2=-314.0D0/6.0D0*(E+5.0D0*B2*BN+5.0D0*B1*BK)
0020      Z1=Y(8)-1.0D0/(G**2)*314.0D0*DERY(7)
0021      Z2=314.0D0*Y(7)+1.0D0/G*314.0D0*DERY(7)
0022      Z3=314.0D0*DERY(7)
0023      T=T1
0024      IF(Z2.GT.0.0) T=T2
0025      SS=-1.0D0/T*Z2*G**2
0026      IF(SS.GT.90.0D0)   SS=90.0D0
0027      Z=G*Z3-T+T*DEXP(SS)
0028      IF(Z.GT.0.0) GO TO 71
0029      VFD=B2*5.0D0
0030      VFQ=B1*5.0D0
0031      GO TO 100
0032   71 VFD=-B2*5.0D0
0033      VFQ=-B1*5.0D0
0034  100 DERY(1)=A(1,1)*Y(1)+A(1,2)*Y(2)+A(1,3)*Y(3)+A(1,4)*Y(4)+
         1A(1,4)*Y(4)*Y(7)+A(1,5)*Y(5)+A(1,5)*Y(5)*Y(7)+A(1,6)*Y(6)+
         2A(1,6)*Y(6)*Y(7)+C11*V*DSIN(Y(8))+C12*RFD/XAFD*VFD
0035      DERY(2)=A(2,1)*Y(1)+A(2,2)*Y(2)+A(2,3)*Y(3)+A(2,4)*Y(4)+
         1A(2,4)*Y(4)*Y(7)+A(2,5)*Y(5)*Y(7)+A(2,6)*Y(6)+A(2,5)*Y(5)+
         2A(2,6)*Y(6)*Y(7)+C21*V*DSIN(Y(8))+C22*RFD/XAFD*VFD
0036      DERY(3)=A(3,1)*Y(1)+A(3,2)*Y(2)+A(3,3)*Y(3)+A(3,4)*Y(4)+
         1A(3,4)*Y(4)*Y(7)+A(3,5)*Y(5)+A(3,5)*Y(5)*Y(7)+A(3,6)*Y(6)+
         2A(3,6)*Y(6)*Y(7)+C31*V*DSIN(Y(8))+C32*RFD/XAFD*VFD
0037      DERY(4)=A(4,1)*Y(1)+A(4,1)*Y(1)*Y(7)+A(4,2)*Y(2)+
         1A(4,2)*Y(2)*Y(7)+A(4,3)*Y(3)+A(4,3)*Y(3)*Y(7)+A(4,4)*Y(4)+
         2A(4,5)*Y(5)+A(4,6)*Y(6)+D11*V*DCOS(y(8))+D12*RFQ/XAFQ*BFQ
0038      DERY(5)=A(5,1)*Y(1)+A(5,1)*Y(1)*Y(7)+A(5,2)*Y(2)+
         1A(5,2)*Y(2)*Y(7)+A(5,3)*Y(3)+A(5,3)*Y(3)*Y(7)+A(5,4)*Y(4)+
         2A(5,5)*Y(5)+A(5,6)*Y(6)+D21*V*DCOS(Y(8))+D22*RFQ*VFQ/XAFQ
0039     DERY(6)=A(6,1)*Y(1)+A(6,1)*Y(1)*Y(7)+A(6,2)*Y(2)+
         1A(6,2)*Y(2)*Y(7)+A(6,3)*Y(3)+A(6,3)*Y(3)*Y(7)+A(6,4)*Y(4)+
         2A(6,5)*Y(5)+A(6,6)*Y(6)+D31*V*DCOS(Y(8))+D32*RFQ*VFQ/XAFQ
0040      IF(X.LT.0.06D0) GO TO 10
0041      DERY(7)=-1.0D0/6.0D0*(XAFD*Y(1)*Y(6)+XAKD*Y(2)*Y(6)-
         1XAFQ*Y(4)*Y(3)-XAKQ8Y(5)*Y(3))-
         21.0D0/6.0D0*DD*314.0D0*Y(7)+
         31.0D0/6.0D0*1.051999999999999*1.0D0
0042      GO TO 25
0043      DERY(7)=-1.0D0/6.0D0*(XAFD*Y(1)*Y(6)+XAKD*Y(2)*Y(6)-
         1XAFQ*Y(4)*Y(3)-XAKQ*Y(5)*Y(3))-
         21.0D0/6.0D0*DD*314.0D0*Y(7)+
```

```
     31.0D0/6.0D0*1.051999999999999D0*2.0D0
 25   DERY(8)=314.0D0*Y(7)
      RETURN
      END

      SUBROUTINE OUTP(X,Y,DERY,IHLF,NDIM,PRMT)
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION DERY(8),Y(8),PRMT(5),AUX(8,8)
      COMMON RFD,XD,XE,XAFD,XAFQ,RE,R,XFFD,XFFQ,XQ,RFQ,XFKD,XFKQ,
     1XKKD,XKKQ,V,XAKD,XAKQ,RKD,RKQ,
      COMMON C11,C12,C13,C21,C22,C23,C31,C32,C33,D1I,D12,D13,D21,D22,
     1D23,D31,D32,D33,X1,X2,X3,X4,DD,G,Z1,Z2,T,Z
      COMMON A(6,6),C(3,4)
      COMMON ZL,XEOLD,REOLD,ZLOLD,VOLD,I,II,J
      COMMON/AA/VFD,VFQ,ET,BN,BK,E,S,Z3
      IF(I/4*4.EQ.I) WRITE(6,88)X,VFD,VFQ,Z,Y(1),Y(3),Y(4),ET,Y(7),
     1DERY(7),Y(8)
 88   FORMAT(1X,1P10D11.3,1P1D13.5)
      I=I+1
      RETURN
      END
```